Life-Cycle Portfolio Choice with Stock Market Loss Framing: Explaining the Empirical Evidence

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Keywords

life-cycle model, portfolio choice, household finance, loss aversion, simulated method of moments

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JEL-Codes: D15, G11, G40, G51

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1   Introduction

Several empirical findings on the actual saving and investment decisions of U.S. households are not well-predicted by standard life-cycle models of the optimal consumption and portfolio choices of a representative investor (pioneered by Cocco, Gomes, and Maenhout 2005). This evidence includes the significant proportion of households not participating in the stock market (Haliassos and Bertaut 1995; Campbell 2006; Gomes and Smirnova 2021), a flat or mildly hump-shaped life-cycle pattern of portfolio shares invested in the stock market by participants (Heaton and Lucas 2000; Ameriks and Zeldes 2004; Catherine 2022), a high level of financial wealth late in life (Love et al. 2009; Lockwood 2018), and a positive relationship between the stock market share and shareholders' financial wealth (Wachter and Yogo 2010).[1] These discrepancies between evidence and model prediction (sometimes referred to as puzzles) grow when important non-financial decisions variables of private households are also included in life-cycle models. For example, including flexible work hours in addition to the consumption and portfolio choice decisions significantly increases the optimal fraction that households would optimally invest in the stock market, compared to the standard-life cycle model (Gomes et al. 2008).

Numerous extensions of the baseline life-cycle model have been proposed to resolve at least one of these puzzles. A large literature offers more refined modeling of the stochastic dynamics of capital market processes (such as stock market crashes), the law of motion for labor income (loss in human capital, cyclical skewness along the business cycle), or the relationship between innovations to stock returns and labor income (Benzoni et al. 2007; Campanale et al. 2015; Michaelides and Zhang 2017; Bagliano et al. 2019; Catherine 2022). A second group of studies include additional background risk such as medical shocks (De Nardi et al. 2010), or the impact of households' pessimistic subjective beliefs by setting the risk premium of equity investments lower than observed in historical data (Dahlquist et al. 2018; Calvet et al. 2022). A third direction taken in prior work assumes significant stock market participation costs for a sizeable set of households, which discourages risky asset holding among those with low financial wealth (Gomes and Michaelides 2005; Kim et al. 2016; Fagereng 2017). Other researchers consider illiquid assets such as housing (Cocco 2005) or life

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[1] This evidence and these puzzles are also existent for other countries as shown by Christelis et al. (2013), Badarinza et al. (2016), Bach et al. (2020), and Fagereng et al. (2020).

annuities (Inkmann et al. 2011) in life-cycle models. A fourth group of studies focuses on extended modeling of household preferences, such as the inclusion of a bequest motive (Ameriks et al. 2011) or non-standard preference specifications (Wachter and Yogo 2010; Pagel 2018; Peijnenburg 2018; Calvet et al. 2022). Although some of these studies combine various model extensions, an approach that jointly replicates the empirical age patterns of private households during both the work and retirement phase for financial wealth holdings, labor supply, stock market participation, and the share of investments in equities, is not as yet available in the literature.

The present paper builds and calibrates a parsimonious life-cycle model that can fit the aforementioned stylized facts. We consider a utility maximizing household with an uncertain lifetime facing labor income and capital market exogenous risks. Each period it decides how many hours to work, how much to consume/save, and how to allocate its portfolio across risky stocks and riskless bonds. The household has recursive preferences over a composite good of consumption and leisure, a bequest motive, and – the main innovation of this paper – a stock market *loss framing* component. Stock market loss framing is defined as an additional disutility the household faces by experiencing investment losses relative to a reference return over the part of the portfolio invested in the stock market. Our approach follows previous work by Barberis and Huang (2009), but it requires only one additional preference parameter instead of two to incorporate loss aversion into a life-cycle model. Accordingly, this strategy offers significant advantages for empirical identification.

The proposed life-cycle model is estimated structurally using a two-stage approach. In the first stage, non-preference parameters (wage rates, capital markets, housing cost, and retirement income) are specified using U.S. data. Notably, we work with an equity risk premium equal to 7.12% as observed in historical data, which is much higher than in many other studies on portfolio choice over the life-cycle (Cocco et al. 2005; Gomes and Michaelides 2005). In the second stage, all preference parameters are estimated using the simulated method of moments (SMM) approach, by targeting the empirical age patterns of observed ratios of stock market participants' portfolio shares in stocks, their financial wealth-to-income ratio, and work hours. These empirical targets are derived from individual-level data from the Survey of Consumer Finances (SCF) and the Panel Study of Income Dynamics (PSID). We explicitly address life-cycle effects by targeting seven age groups for each of the variables, from the beginning of a

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household's working life (i.e., at age 25) to retirement (up to age 80). In addition, we examine two subgroups of the population, namely stockholders and non-stockholders.

To gain a deeper understanding of the proposed loss-framing preferences, we first focus our analysis on the important subset of households which participates in the stock market. We demonstrate that our structural life-cycle model with loss-framing preferences can simultaneously fit the three target variables remarkably well, namely the stock share, wealth-to-income-ratio, and work hours. In the estimation, 20 of 21 moments predicted by the model lie within or near the 95% confidence bounds of their empirical counterparts. The mean relative error (MRE) between model prediction and empirical target over all 21 moments is only 3.60%. Our parameter estimates for relative risk aversion, time preference, and intertemporal elasticity of substitution are also plausible and in line with other life-cycle studies. Moreover, estimates for both the loss-framing parameter and the bequest parameter are significantly above zero.

Second, a decomposition of the preference components according to their influence on results shows that the model fit with stock market loss-framing is far superior to other structural models lacking such a component. As such, compared to the data, models with traditional CRRA preferences (over consumption and leisure) including a bequest motive predict lower work hours, higher conditional stock market shares, and lower financial wealth-to-income ratios, especially for the middle-aged and older groups. For the CRRA model, the average deviation of predicted model moments from empirical targets is 20.71%, nearly six times larger than our model with stock market loss framing (3.60%). Models with Epstein-Zin preferences generate better matching results for conditional stock market shares, but they still underestimate work hours and financial wealth-to-income ratios; their average deviation of model moments vis-à-vis empirical counterparts is about five times larger than our loss-framing model (16.71% versus 3.60%). Further, many other models exhibit implausible extreme parameter estimates for the time discount factor, the coefficient of relative risk aversion, or the leisure preference.

Third, while our loss framing model is well suited to explain the empirical mildly hump-shaped and relatively low ratio of stock investments for the equity holder subgroup, it cannot additionally explain the prevalence of non-participation in the stock market in the overall population. To achieve this, the life-cycle model with loss framing must be extended. In particular, we show that a structural model with a single-representative agent and per-period stock market participations costs, in conjunction with loss-framing preferences, does generate

a reasonable match for four target variables over all seven age groups considered. The mean relative error of predicted 28 model moments in relation to their empirical targets is only 9.6%.

Fourth, further improvement can be achieved if households are ex-ante heterogeneous with respect to participation costs and preferences. A loss-framing model with a representative agent for each of three subgroups (stockholders, non-stockholders, and blended) explains the data for the overall population much better than a simpler model with only one representative agent (MRE equals 4.3% versus 9.6%). Furthermore, consistent with the data, this richer approach predicts that stock-market participants are significantly wealthier than households not investing in equities. Finally, the model is able to produce increasing stock market participation rates and conditional equity shares with growing levels of financial wealth, as observed in the data.

Our study contributes to three strands of prior literature. First, we build on studies which structurally estimate preference parameters to match key life-cycle target variables.[2] Articles from the precautionary saving literature studies such as De Nardi et al. (2010) target household wealth patterns using structural estimation, especially the slow wealth decumulation of the elderly. French (2005) and Abe et al. (2007) also include work hours as target variable in the structural estimation of their life-cycle model. Yet such studies set aside the portfolio selection decision by permitting households to invest in only one asset. Research addressing the portfolio choice problem includes Inkmann et al. (2011), Fagereng (2017), Dahlquist et al. (2018), Pagel (2018), Bonaparte et al. (2021), and Catherine (2022). These studies include stock market participation and/or the share of financial assets invested in equities (conditional on participation) in their structural estimation. While some also target wealth accumulation profiles, they do not include non-financial decision variables such as work hours. Consequently, our life-cycle model combines these two approaches to better match simultaneously the age-profiles on stock market participation rates, conditional stock share, financial-wealth-to income ratio, and work hour profiles. In particular, and in contrast to most studies, we include relatively high age ranges as targets in the structural estimation, and thus we address household behavior during both their work and retirement phases. All this is done with a minimum of additional state variables.

A second strand of literature to which our work is related studies the implications of loss aversion on optimal portfolio allocations. For example, Blake et al. (2013) uses such a model

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[2] This literature is heterogeneous as some studies only match one moment across all age groups while other studies also include other non-preference parameters such as stock market participation costs in the structural estimation.

to determine the optimal investment strategy for participants in a defined contribution pension plan. Yet that study focuses on only the retiree's final wealth at retirement, and not consumption over the entire life cycle.[3] Other studies develop life-cycle models with a narrow framing preference specification as suggested by Barberis and Huang (2009), assuming that households derive utility not only from consumption but also from periodic gains and losses of their risky asset holdings. Recent work by Calvet et al. (2022) uses such an approach to explain the high demand for investment products with guaranteed returns in the Swedish market. An earlier paper by Chai and Maurer (2012) attempts to derive the reference point from investors' historical return experiences in the context of a life-cycle model. We contribute to this literature by showing theoretically that the two-parameter approach of Barberis and Huang (2009) can be reduced to a single preference parameter. In addition, such papers set the preference parameters used in their models along pre-specified values from the literature. By contrast, we determine the preference parameters endogenously as a result of a structural estimation procedure.

Last, our work also extends the life-cycle literature addressing ex-ante as well as ex-post heterogeneity of households in the overall population (Gomes and Michaelides 2005; Calvet et al. 2021; Gomes and Smirnova 2021). In particular, we show that a structural life-cycle model with heterogeneous preferences for three important subgroups of the total population, namely stock owners, bond owners, and a mixed group, can explain the data remarkably well.

The remainder of the paper is as follows. Section 2 presents the life-cycle model. Section 3 explains the calibration and structural estimation. Section 4 displays the analysis for the model for stockholders. Section 5 outlines the analysis for the model for the entire population. Section 6 presents the analysis for the heterogeneous agent model. Section 7 concludes.

2   Life-Cycle Model

Our overall objective is to match a rich dataset of key financial and non-financial household decisions by age, to the predictions of a structurally estimated life-cycle model which allows us to estimate household preferences. The model itself is an attempt to balance parsimony along with important financial and non-financial variables using a state-of-the-art life-cycle model. The model is deliberately chosen to be parsimonious with respect to the

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[3] This study is also related to studies that develop dynamic portfolio choice models with narrow framing and loss aversion that do not include other typical life-cycle model components such as labor income. De Giorgi and Legg (2012) extend the framework of Barberis and Huang (2009) by additionally including probability weighting into the narrow-framing component, while Guo and He (2021) extend the framework by scaling gains and losses proportionally to the certainty equivalent of the total utility.

number of state variables, as we solve it numerous times during the structural estimation and thus the solution to the model must be computationally tractable.

2.1 Preference Specification

We work in discrete time and assume that the representative U.S. household faces an uncertain lifetime and makes annual decisions from age 25 ($t_0 = 1$) to age 100 ($T = 76$) on consumption, leisure, and portfolio allocation to stocks and bonds. The household has rational expectations and receives utility from consumption, leisure, bequest, and an additional stock market loss framing component. The overall preference structure is defined as follows:

$$V_t = \left\{ (1-\beta)(C_t L_t^\alpha)^{1-\frac{1}{\psi}} + \beta \left[\left(\mathbb{E}_t \left[p_t V_{t+1}^{1-\gamma} + (1-p_t)b \left(\frac{W_{t+1}}{b} \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}} \right. \right.$$

$$\left. \left. + p_t \Lambda \mathbb{E}_t[\min(G_{t+1}, 0)] \right]^{1-\frac{1}{\psi}} \right\}^{1-\frac{1}{\psi}}, \tag{1}$$

where the performance from investments in the stock market S_t generating uncertain returns R_{t+1} relative to a benchmark return R_b is given by:

$$G_{t+1} = S_t(R_{t+1} - R_b). \tag{2}$$

Accordingly, the household has recursive preferences of the Epstein-Zin (1989) type defined over a composite good comprised of non-durable consumption C_t and leisure L_t (normalized as a fraction of total available time), which themselves are governed by a modified Cobb-Douglas function with leisure parameter $\alpha > 0$. Furthermore, $0 < \beta < 1$ denotes the subjective discount factor, $\psi > 0$ represents the elasticity of intertemporal substitution (EIS), and $\gamma > 1$ denotes the coefficient of relative risk aversion. The probability of surviving to period $t + 1$ conditional on being alive in period t is denoted by p_t. The household further receives future utility by financial wealth W_t transferred to the next generation in case of death; the parameter $b \geq 0$ determines the strength of the bequest motive. The last preference component is a modification of the narrow-framing component with embedded loss aversion proposed by Barberis and Huang (2009), where we concentrate on the fall in utility resulting

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from stock losses.[4] Specifically, the household experiences additional disutility from the expected shortfall on stock investments below a predefined benchmark R_b and the parameter $\Lambda \geq 0$ measures the strength of this preference component.

Five features make this preference specification attractive in the life-cycle context. First, it is dynamically consistent, which implies that dynamic programming techniques can be used for solving models with such preferences.[5] The stock market loss framing component does not alter this property as the household knows at each time step how it will frame any future losses. Second, the whole preference specification is homothetic, which is a desirable property for preferences as it allows for normalization by state variables to reduce the computational burden of solving the model. Third, none of the elements comes at the cost of an additional state variable, making the life-cycle model parsimonious and allowing the inclusion of other state variables from a numerical point of view. Fourth, it is straightforward to eliminate certain parts of the preference specification, including the loss-framing part (by setting $\Lambda = 0$), such that it collapses to a classic preference specification often used in the life-cycle context and thus enables a distinct analysis of effects. And fifth, in contrast to Barberis and Huang (2009), our approach requires only one parameter Λ to capture loss preferences, which substantially facilitates the dimensionality of the matching procedure in the structural estimation. We illustrate in Appendix A how our loss-framing parameter Λ is related to the Barberis and Huang (2009) parameters for narrow framing and loss aversion and show that these can be translated into one another.

2.2 Capital Market and Asset Dynamics

The household can invest in two financial assets, a risk-free bond and a risky asset represented by an investable stock index. The dollar amount invested in bonds and stocks at time t is denoted by B_t and S_t respectively. To avoid modeling potential household insolvency, we rule out household borrowing and stock short-selling. The bond yields a constant real annual gross return R_f, also the benchmark to determine losses from equity investments entering the utility function in equation (2). Investments in risky stocks have an uncertain gross return R_t that follows a stochastic process:

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[4] It should be noted that the specification of the loss framing part in (1) is consistent with Bawa and Lindenberg's (1977) first order Lower Partial Moment $LPM_1 = E[\max(S_t R_b - S_t R_{t+1}, 0)] = -E[\min(S_t R_{t+1} - S_t R_b, 0)]$ and Fishburn's (1977) mean-risk analysis, in which risk is measured by the expected shortfall below a target return.
[5] This is the case as the Epstein-Zin preferences are themselves a case of Kreps-Porteus (1978) preferences, which are dynamically consistent as illustrated by Kreps and Porteus (1979).

$$\log(R_t^{gross}) = \mu + \eta_t, \tag{3}$$

where μ is the drift and $\eta_t \sim N(0, \sigma_\eta^2)$ is an innovation component, assumed to be independent and identically normally distributed. In sensitivity analysis, we allow stock returns to follow a Markovian regime-switching process with different drift and volatility parameters to model a disaster event.

The household faces three types of costs for capital market investments: first, yearly returns on stocks and bonds are subject to a linear capital gains tax t_c.[6] Second, a yearly management fee κS_t proportional to the dollar amount invested in stocks, constitutes a fixed reduction of the expected annual (pre-tax) stock return. Such a fee is usually charged by asset managers and reflects their administrative and distribution costs. Therefore, the return on equities net of management fee is given by $R_t = R_t^{gross}(1 - \kappa)$. These elements result in the following dynamics for financial wealth:

$$W_{t+1} = S_t R_{t+1} + B_t R_f - t_c \max(S_t R_{t+1} + B_t R_f - S_t - B_t, 0). \tag{4}$$

Third, the household faces costs for stock market participation. As pointed out by Vissing-Jørgensen (2002), there are several sources of such costs. Some are pecuniary in nature, such as fees for a broker account or paying for financial advice. There are also opportunity costs, such as the requirement to devote time and mental resources for gathering and processing information about market conditions and evaluating risk-return characteristics (Ahn et al. 2011; Kim et al. 2016). While there may be an initial one-time entry cost of stock participation (Gomes and Michaelides 2005), it will also still be necessary for investors to expend resources each period to evaluate and to respond to changing financial market conditions. Here we follow Fagereng et al. (2017) and assume that investments in the stock market incur fixed per-period participation costs ϕ_t; we allow such costs to depend on age, and they may also be heterogeneous across different types of investors.

2.3 Labor Income

During the working life until period K, the household can allocate the fraction N_{t-1} of its available yearly time budget Υ (in hours) to work paying an uncertain hourly wage rate. Total gross labor income follows the following stochastic process:

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[6] For simplicity, we do not distinguish between the different taxation of dividends and capital gains on stocks nor do we allow to carry forward losses into subsequent periods. Instead, we assume that all investment income (if positive) is taxed with the same tax rate at the source.

$$Y_t^{gross} = N_{t-1}\Upsilon exp\big(w(t, N_{t-1})\big)P_t\varphi_t \qquad \forall\, t \leq K, \tag{5}$$

where $w(t, N_{t-1})$ denotes the logarithm of the wage rate in time period t, which is a deterministic function of age as well as the work time, and φ_t is an idiosyncratic temporary wage shock. P_t is the level of permanent income which develops according to the following dynamics:

$$P_t = P_{t-1}\vartheta_t, \tag{6}$$

where the innovation ϑ_t to the permanent component is further decomposed into an aggregate component ξ_t and an idiosyncratic component ω_t:

$$\vartheta_t = \xi_t\omega_t. \tag{7}$$

All three wage shocks are serially independent and identically log-normally distributed, i.e. $\log(\varphi_t) \sim N\big(-0.5\sigma_\varphi^2, \sigma_\varphi^2\big)$, $\log(\xi_t) \sim N\big(-0.5\sigma_\xi^2, \sigma_\xi^2\big)$, and $\log(\omega_t) \sim N\big(-0.5\sigma_\omega^2, \sigma_\omega^2\big)$, and they are assumed to be uncorrelated with each other. The aggregate shock to permanent income ξ_t may be correlated with the shock to the stock return η_t.

Similar to Gomes et al. (2008) and Hubener et al. (2016), we assume that net labor income, which the household can use for non-durable consumption and saving, is equal to gross labor income after the deduction of proportional age-dependent housing costs $h(t)$ and a proportional labor income tax rate t_l. The resulting net labor income is then defined as follows:

$$Y_t = (1 - h(t))(1 - t_l)Y_t^{gross}. \tag{8}$$

This modeling of housing costs and labor income taxes is common in the life-cycle literature (see for example Gomes et al. 2008), especially because it does not require additional state variables.[7]

2.4 Retirement Income

After retirement at the exogenous time $K + 1$, the household receives lifelong Social Security benefits based on a fraction ζ (the replacement rate) of average lifetime earnings $\bar{Y}$:

$$Y_t^{gross} = \zeta\bar{Y} \qquad \forall\, t > K. \tag{9}$$

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[7] See Cocco (2005) and Yao and Zhang (2005) for life-cycle models that include housing as an additional asset and Zhou (2012) and Horneff et al. (2019) for life-cycle models that include the federal labor income tax according to the official U.S. rules.

Following Chai et al. (2011), the average lifetime earnings level is approximated by the product of weighted deterministic wages during the working life, the average yearly work effort $\bar{N}Y$, and the permanent labor income P_K at the retirement age:[8]

$$\bar{Y} = \frac{\sum_{t=1}^{K} \bar{N}Y\exp\big(w(t, \bar{N})\big)}{K} P_K. \tag{10}$$

To arrive at net retirement income available for consumption or saving, we further adjust gross retirement income by deducting age-dependent proportional housing costs $h(t)$ and a proportional retirement income tax t_r, which is different than during the working life. In addition, we shrink gross retirement income by the proportional factor $1 - \lambda$, where λ represents the average portion of retirement income the household must spend on medical costs, and ε_t represents a transitory retirement income shock, where $\log(\varepsilon_t) \sim N(-0.5\sigma_\varepsilon^2, \sigma_\varepsilon^2)$. The latter captures the risk of out-of-pocket medical expenses during retirement similar to Pang and Warshawsky (2010). Consequently, net retirement income is given as follows:

$$Y_t = (1 - h(t))(1 - t_r)(1 - \lambda)Y_t^{gross}\varepsilon_t. \tag{11}$$

2.5 Optimization Problem

The household maximizes its value function, equivalent to the preference specification in equation (1), for each time step t. We exploit the fact that the value function is homothetic and normalize all pecuniary variables by permanent income P_t, which enables us to drop permanent income as a state variable. Lowercase variables represent the normalized counterparts of uppercase non-normalized variables. The household chooses the control variables consumption c_t, investment in stocks s_t, investment in bonds b_t, and work hours (as a fraction of available time) N_t. Normalized cash on hand x_t at a certain point in time represents the only state variable,[9] where cash on hand is defined as the sum of financial wealth and net labor or retirement income:

$$x_{t+1} = w_{t+1} + y_{t+1}. \tag{12}$$

We solve the optimization problem for two cases. In the case *without stock market participation costs*, the optimization problem reads as follows:

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[8] The modeling of the average lifetime earnings is a reasonable approximation of the U.S. Social Security rules, which use the Average Indexed Monthly Earnings (AIME) to calculate a household's retirement benefit, where the AIME is defined as the mean of the household's 35 highest years of indexed earnings up to age 60. Yet tracking yearly earnings requires a least one additional state variable, which increases the computational effort substantially.
[9] We also consider one model variant in which the stock process follows a two-dimensional Markov regime-switching process. For that model the current financial regime constitutes a discrete state variable.

$$\max_{c_t, s_t, b_t, N_t} v_t(x_t). \tag{13}$$

The constraints to which the household must adhere include the budget constraint:

$$x_t = c_t + s_t + b_t, \tag{14}$$

the non-negativity constraints for consumption as well as stock and bond holdings:

$$c_t, s_t, b_t \geq 0, \tag{15}$$

and the work budget constraint:

$$\tilde{N} \geq N_t \geq \underline{N}, \tag{16}$$

where $\tilde{N}$ and $\underline{N}$ are the maximum and minimum of available time (normalized as a fraction of total available time) the household can allocate to work, respectively. During retirement, the household must not work, which is why constraint (16) is binding at the lower bound:

$$N_t = \underline{N} \qquad \forall\, t > K. \tag{17}$$

The work decision has direct implications for household leisure which itself enters the preference specification:

$$L_t = 1 - N_t. \tag{18}$$

In the case *with stock market participation costs*, we extend the previous model and assume that an age-dependent fixed amount ϕ_t must be paid to participate in the stock market, following Fagereng et al. (2017). If the household has enough cash on hand to cover this participation cost as well as to keep consumption and stocks investment positive, it optimizes two problems, one with and one without stock market participation. The optimization problem if the household decides to participate in the stock market is defined as follows:

$$\max_{c_t, s_t, b_t, N_t} v_t(x_t^s) \tag{19}$$

with the budget constraint:

$$x_t^s = c_t + s_t + \phi_t + b_t. \tag{20}$$

The optimization problem if the household decides to not participate in the stock market is defined as follows:

$$\max_{c_t, b_t, N_t} v_t(x_t^b) \tag{21}$$

with the budget constraint:

$$x_t^b = c_t + b_t. \tag{22}$$

The optimization problem is then as follows:

$$v_t = \max\left(v_t(x_t^s), v_t(x_t^b)\right). \tag{23}$$

The household finally decides whether to participate in the stock market by comparing the value function of participation and non-participation, and selects the option with the higher value:

$$\mathbb{1}_{\{S_t>0\}} = \begin{cases} 1 & \text{if } v_t(x_t^s) \geq v_t(x_t^b) \\ 0 & \text{if } v_t(x_t^s) < v_t(x_t^b) \end{cases}. \tag{24}$$

If the participation costs ϕ_t are greater than the cash on hand x_t, an investment in stocks is not possible. In this case the optimization problem is equivalent to equations (21) and (22).

2.6 Numerical Solution Methods

We solve the optimization problem using discrete-time dynamic programming, solved recursively through time via backward induction. Given that the preference specification is homothetic and the permanent income component P_t follows a random walk, we normalize each pecuniary variable by permanent income P_t and thus can remove permanent income as a state variable in the optimization problem. We discretize the continuous state variable normalized cash on hand x_t on an equidistant 35-point log-grid with a lower bound of 0.6931 and an upper bound of 7.6014, where the scale of the grid points is in thousands of U.S. dollars. The expectation of the multivariate log-normally distributed random variables is computed using Gauss-Hermite quadrature with nine quadrature nodes per dimension. To evaluate the value function at points for normalized cash on hand that do not lie on the grid, we use a cubic spline inter- and extrapolation. We further use an interior-point algorithm to solve the constrained non-linear optimization problem. After solving the optimization problem and obtaining the corresponding policy functions, we simulate the model using Monte Carlo simulation with 100,000 paths, where each path represents one life-cycle scenario of the household. For the simulation, we assume that the household starts with some work hours and financial wealth. To introduce some ex-ante heterogeneity into our economy, we equip the household with differing initial financial wealth in each simulation path. The initial financial wealth is defined as a multiple of the household's first year gross income, drawn from a generalized Pareto distribution. First-year gross income is based on 40 hours of work per week for each household. The ex-post heterogeneity is then given by the realizations of all shocks during the simulation.

3 Estimation and Calibration

For calibrating and estimating the parameters of our model, we adopt a two-stage approach similar to that in Gourinchas and Parker (2002), De Nardi et al. (2010), French and Jones (2011), and Laibson et al. (2015). Our approach in the first stage is to estimate as many model parameters as possible and thus minimize discretionary decisions, and our estimation methods follow standard procedures in the literature. Therefore, in the first stage, we estimate and calibrate parameters related to labor and retirement income, housing costs, capital market returns, and mortality rates using U.S. data, given that these can be identified without explicitly solving the model. In the second stage, we structurally estimate preference parameters and stock market participation costs given the first-stage parameters using the simulated method of moments (SMM) with respect to several empirical target variables.

3.1 First-Stage Parameters

All calibrations and estimations rely on up-to-date U.S. data. The longevity of our representative households is governed by survival probabilities p_t obtained from the Human Mortality Database (HMD) period life table 2017. Table 1 gives an overview of all first-stage model parameters, while more details on data, estimation methods, and results are displayed in Appendix B.

--- Table 1 here ---

The risk-free rate and stock return parameters are estimated using the 3-Month T-Bill and the S&P 500 total return stock index for the sample period February 1970 to July 2020, respectively, deflated by the monthly CPI for all urban customers using as a base month July 2020. This generates a yearly riskless rate of $R_f = 1.0077$, which also serves as the benchmark to determine relevant losses of stock investments entering the utility function. The expected gross stock return is $\mathbb{E}[R_t^{gross}] = 1.0789$ and the standard deviation is $\text{Std}(R_t^{gross}) = 0.1688$. Consequently, our estimate for the equity risk premium of $\mathbb{E}[R_t^{gross} - R_f] = 0.0712$ is substantially higher compared to that used in other life-cycle studies, which typically assume a risk premium of around 0.04 (Cocco et al. 2005; Gomes and Michaelides 2005); using a lower premium of course makes stock market investment less attractive.

Our estimation of the gross wage process parameters uses data from the 1975 to 2017 waves of the Panel Study of Income Dynamics (PSID). Following Carroll and Samwick (1997) and Cocco et al. (2005), we evaluate the deterministic and the stochastic wage components; the

contemporaneous correlation of the logarithm of the aggregate component of the stochastic permanent wage with the innovation of the logarithm of the stock return η_t follows Campbell et al. (2001) and Cocco et al. (2005). Resulting estimates are in line with values in the literature. Total gross labor income is then defined as the exponential of the log wage rate process times yearly waking hours $\Upsilon = 5200$, which is itself the result of the product of 52 weeks and an assumed 100 waking hours per week the household can allocate at the maximum to leisure. The fraction N_t determines the percent of waking hours the household can allocate to work, with $N_t \in [\underline{N}, \tilde{N}]$. We set $\underline{N} = 0$ and $\tilde{N} = 0.6$, which results in a range of 0 to 60 hours work per week. We assume that age 67 is the sole retirement age, which corresponds to time period $K + 1 = 43$. Estimation of the retirement parameters, namely the benefit replacement ratio, the out-of-pocket medical expenditures, and the stochastic component of net retirement income rely on the PSID and procedures similar to Love (2010). Age-dependent housing costs are estimated with PSID data following Gomes and Michaelides (2005) and Love (2010).

The proportional management fee for stock investments is $\kappa = 0.0018$, which is equal to the difference of the average of pre- and post-expense return of exchange traded funds reported by Elton et al. (2019). The age-dependent stock market participation costs $\phi_t = F_t \cdot \chi$ per-period is the product of the fixed participation costs and an age-dependent scaling factor F_t. The scaling factor is determined using the American Time Use Survey (ATUS) microdata from the Bureau of Labor Statistics, such that at age 66 the ratio is equal to one. The function for the scaling factor is equal to $F_t = 0.0152(t + 24)$. The uniform proportional capital gains tax is set to $t_c = 0.15$, the proportional labor income tax is set to $t_l = 0.3$, and the proportional retirement income tax is set to $t_r = 0.1$.

Household initial financial wealth is defined as a multiple of the household's first-year gross income, drawn from a generalized Pareto distribution that fits the financial wealth-to-income ratio of households aged 18 to 25 from the Survey of Consumer Finances for the relevant (sub-)groups of the population.[10] The first-year gross income is based on 40 weekly work hours for each household.

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[10] The financial wealth-to-income ratio is defined as the ratio of total financial wealth to annual labor or retirement income. The exact definitions are explained in detail in the subsequent section, where the empirical targets for the structural estimation are discussed.

3.2 Structural Estimation

3.2.1 Empirical Targets

Despite its sparsity, our life-cycle model generates several of the target variables of interest for our structural estimation. We choose to investigate the household's portfolio allocation, savings, and labor supply decisions, which are the key household choices. These variables are approximated by the decision to hold any stocks, the fraction of financial wealth invested in stocks, the ratio of financial wealth to labor and retirement income, and the fraction of available time devoted to work, respectively. Furthermore, in terms of wealth accumulation, we distinguish between stockholder households, non-stockholder households, and blend of both types in the entire population.

We use two datasets for estimating the empirical targets. The data for all portfolio and wealth-related targets are obtained from the Survey of Consumer Finances (SCF), which is a triennial survey on the financial assets of U.S. households. Data for work hours are obtained from the Panel Study of Income Dynamics (PSID), which we use to estimate the labor and retirement income-related parameters in the first stage. Details on the exact data and definitions of the empirical targets are displayed in Appendix C.

Further, each variable is grouped into seven age buckets. For all variables, the starting age is 25; the ending age is 80 for the SCF-based variables, and age 66 for the PSID-based variable. In order to give each targeted variable the same weight, we use the same number of age groups for each variable.[11] This is supported by the greater data availability compared to single-age groupings, and further it has the advantage of providing some data smoothing. Moreover, this approach reduces estimate bias (Newey and Windmeijer 2009).

Definitions of the targeted empirical variables are then as follows. Stock market participation is an indicator variable that equals one if the total dollar stock allocation is above zero, and zero otherwise. The stock share conditional on stock market participation is the fraction of financial wealth allocated to stocks, given a non-zero allocation to stocks. The financial wealth-to-income ratio is the ratio of total financial wealth to annual labor or retirement income. The financial wealth-to-income ratio for stockholders is the ratio given a non-zero allocation to stocks, while the financial wealth-to-income ratio for non-stockholders

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[11] The decision to use seven age groups for each variable is grounded in the modeling decision of the life-cycle model. We have 42 integer ages for the working phase (age 25 to 66), this results in seven groups over each of six consecutive years of life. This integer factorization must also be applied to the wealth-related SCF-variables with starting age 25 and ending age 80, which results in seven age buckets over each of eight consecutive years of life.

is the ratio given a zero allocation to stocks. We further trim the data by deleting observations with negative financial wealth-to-income ratios and those above 100 for more robust estimates. The share of weekly work hours is defined as the average weekly work hours reported in the PSID divided by our assumed 100 weekly waking hours.[12]

To estimate the desired life-cycle effects, we must disentangle age, time, and cohort effects which together are perfectly collinear. To this end, we rely on the method developed by Deaton and Paxson (1994), by regressing the empirical target $y^{j,data}$ on a full set of age group dummies as well as cohort dummies and wave/time dummies. For all variables relying on SCF data, we weight both the independent and the dependent variables by a scaled wave weight that ensures that the sum of all weights is equal across the waves. The regression for each variable observation of household i is then as follows:

$$y_{i,a,c,t}^{j,data} = \beta_a D_a + \beta_c D_c + \beta_t D_t + \epsilon_{i,a,c,t}, \tag{25}$$

subject to:

$$\sum_t \beta_t (t - \bar{t}) = 0, \tag{26}$$

where D_a, D_c, and D_t are dummies for age, cohort, and time (i.e. waves), respectively. The constraint that the sum of the wave coefficients times the demeaned wave year is equal to zero ensures that cohort effects capture any time trend.

Estimation results are displayed in Table 2. In general, all regression coefficients are highly significant, given the large sample sizes. The stock market participation rate is hump-shaped with values between 0.52 and 0.65. The conditional stock share shows a mildly hump-shaped pattern over the life cycle, lying between 0.38 and 0.46. The financial wealth-to-income ratio for stockholders is strictly increasing and convex in age, while the financial wealth-to-income ratio for non-stockholders is also increasing in age but flattens out for the oldest age groups. The financial wealth-to-income ratio for the entire population is also increasing in age, fairly linearly. Accordingly, there is a large difference in wealth accumulation between stockholders, households that hold only bonds, and the entire population, illustrating the usefulness of distinguishing between these investor groups. The work hours share is stable over the life-cycle with an average share of about 0.42, which drops slightly for the oldest age group.

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[12] It would also be possible to condition the work hours share on stock market participation, but the PSID includes information on combined stock ownership in retirement and non-retirement accounts in only three waves (1984, 1989, and 1994). Moreover, in more recent waves, stock ownership is reported only for non-retirement accounts, which heavily underestimates the population stock market participation pattern.

--- Table 2 here ---

3.2.2 Methodology

One model run of the SMM estimation proceeds as follows. First, we specify a set of second-stage parameters. Next, for this parameter combination, the life-cycle model is optimized and then simulated given the first-stage parameterization. Third, the moments of the target variables are computed using the simulation results for that parameter combination. Fourth, the simulation-implied moments are compared to those of the empirical targets using a distance function, which determines the goodness-of-fit of the respective parameter combination. In the following, we give a brief overview of the methods (further details on the exact methods for each component of the structural estimation are displayed in Appendix D).

The vector $\boldsymbol{\theta}$ comprises all preference parameters contained in equation (1) as well as stock market participation costs and is defined as follows:

$$\boldsymbol{\theta} \equiv [\alpha \ \beta \ \gamma \ \psi \ b \ \Lambda \ \chi]. \tag{27}$$

The loss benchmark parameter R_b is set equal to R_f, as the other economically reasonable value of one would be the same as upscaling the loss-framing parameter. The SMM estimator is then the resulting vector of parameters that minimizes this distance function, which is defined as the squared percentage deviation of the difference in the vector of simulated model moments from the data moments:

$$\boldsymbol{\theta}_{SMM} = \arg \min_{\theta} \left[\frac{\boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}) - \boldsymbol{m}(\boldsymbol{y}^{data})}{\boldsymbol{m}(\boldsymbol{y}^{data})} \right]^{\top} \boldsymbol{W} \left[\frac{\boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}) - \boldsymbol{m}(\boldsymbol{y}^{data})}{\boldsymbol{m}(\boldsymbol{y}^{data})} \right], \tag{28}$$

where $\boldsymbol{m}(\cdot)$ denotes the moment vector of the targeted variables. We use the percentage deviation of the moments as a distance measurement, given the different scale of the target variables. Furthermore, the weighting matrix $\boldsymbol{W}$ is equal to the identity matrix $\boldsymbol{I}$ as we combine moments from two different data sets, namely the SCF and the PSID. Consequently, the typical modeling approaches cannot be applied, such as using inverse of the variance-covariance matrix of the actual data moments (Gourieroux et al. 1993) or bootstrapping the actual data (Hall and Horowitz 1996). Standard errors are calculated in line with Fagereng et al. (2017) by bootstrapping the simulated data.

To determine the best-fitting parameter combination, first we perform an exhaustive discrete parameter grid search.[13] The upper and lower values for the constraint set of the multidimensional grid are extrapolated values of previously-used values in the literature. This approach ensures that our parameter grid construction is consistent with respect to each parameter and some extent literature.[14] Given the high dimensionality of the parameter space and consequently the total number of possible parameter combinations, we extract our parameter values using quasi-random numbers from a Sobol sequence, which has the advantage of constructing a relatively even distribution of the parameter combinations in each dimension. Furthermore, this approach has the convenience that one can extract a variety of empirical target combinations from one combination run and utilize large-scale parallelization techniques over the parameter combinations. In the second step, we use the three best-fitting parameter combinations for each empirical target combination from the first step, to run for each, a local optimization using a direct search algorithm where the respective parameter combination serves as starting point. The best-fitting parameter combination of all runs in the second step is then considered our global optimum and hence the structural estimate for the respective target moment combination.

Our assessment of the goodness-of-fit of the model is performed via two criteria, namely the numerical value of the criterion function value at the best-fitting parameter combination:

$$f(\boldsymbol{\theta}_{SMM}) = \left[\frac{\boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM}) - \boldsymbol{m}(\boldsymbol{y}^{data})}{\boldsymbol{m}(\boldsymbol{y}^{data})}\right]^{\top} \boldsymbol{I} \left[\frac{\boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM}) - \boldsymbol{m}(\boldsymbol{y}^{data})}{\boldsymbol{m}(\boldsymbol{y}^{data})}\right] \quad (29)$$

and the mean relative error (MRE), which is defined as the mean of the absolute percentage deviation of the targeted model moments and the data moments, i.e.:

$$MRE(\boldsymbol{\theta}_{SMM}) = \frac{1}{n}\sum_{i}^{n}\left|\frac{m_i\left(y^{j,model}|\boldsymbol{\theta}_{SMM}\right) - m_i\left(y^{j,data}\right)}{m_i(y^{j,data})}\right|, \quad (30)$$

where n denotes the total number of targeted moments, and $m_i\left(y^j\right)$ denotes the i-th moment of target variable j. The latter metric enables us to also compare the fit between models with a different number of targeted moments.

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[13] A discrete parameter grid search for matching data moments in life-cycle models is also used, for example, by Love (2010) and Hugonnier et al. (2013).

[14] We mainly include literature-based values that are not structurally estimated. We also exclude extreme values that do not make sense in our context, and we do not use natural boundaries as starting values in order to allow the extrapolation to reach values in both directions without being immediately bounded. The literature-based parameter values, their sources, and the final extrapolated values can be found in the Appendix D.

4 Analysis for Stock Market Participants

To illustrate the proposed loss-framing preferences, we initially focus on the important subset of households that does participate in the stock market. In particular, we do not target stock market participation rates, but we do include age profiles of three target variables, namely the share of financial wealth invested in equities, the financial wealth-to-income ratio, and work hours. This allows us to include only preference parameters in our structural estimation. Furthermore, this enables us to isolate the impact of our loss-framing concept without interference from other model components and variables known to have an effect on stock market participation (such as participation costs). Nevertheless, below we show that the loss-framing preference specification alone does not discourage households to completely avoid the stock market. In Sections 5 and 6, this simplification is abandoned, so we can directly address the limited stock market participation of U.S. households.

4.1 Estimation Results

This section presents our structural estimation results for three different target moment combinations (see Panel A of Table 3). In the first setting, depicted in column 1, only the shares of financial wealth invested in the stock market for the seven age groups are used as targets. Subsequently, the work hours (column 2) are also included. In the third setting (column 3), the empirical moments of all three variables, namely the conditional stock share, the work hours share, and the financial wealth-to-income ratio across the seven age groups (i.e. 21 moments overall) are considered as target variables for the structural estimation procedure. The goal of these successive evaluations is to highlight the importance of estimating simultaneously all three target variables, rather than limiting the structural estimation to only one or two targets. Optimal preference parameters with the best fit of the moments generated by the model relative to the empirical moments are listed in Panel B of Table 3, while the model-implied moments are contrasted to empirical moments in Figure 1.

The first setting which targets only the conditional stock share data shows a nearly perfect fit of the empirical moments. The mean relative error of the seven moments generated from the model relative to their empirical counterparts is only 1.32%. Furthermore, Figure 1 shows almost no visible difference between the empirical moments (solid line) and the model values (dashed line). Using the standard errors documented in Table 2, all moments generated by the

model are well within the 95% confidence interval of their empirical counterparts.[15] The loss-framing parameter Λ is significantly different from zero with an estimated value of 0.0201.[16] Therefore, it can be concluded that the consideration of an aversion to losses resulting from equity investments does play a key role in explaining the data so well.

Nevertheless, a review of the optimal values generated for the other preference parameters reported in column (1) of Table 3 Panel B illustrates a weakness of this approach. On the positive side, the preference parameters for leisure α, relative risk aversion γ, and the strength of the bequest motive b are reasonable and highly statistically significant, comparable to those in other life-cycle studies.[17] Yet the estimated time discount factor β and the EIS ψ, respectively 0.8417 and 0.2677, are far too low, and far from their empirical counterparts (see Figure 3 Panels a.2 and a.3). Compared to the data, these households work fewer hours and accumulate much less financial wealth, given their low discount factor.

--- Table 3 here ---

Our second estimation combination in column (2) of Table 3 Panel B targets the conditional stock and work hours share, while the financial wealth-to-income ratio continues to be excluded from the structural estimation procedure. Resulting parameter estimates are not too different from those in the previous case, as the discount factor, the EIS, and the bequest parameter increase in value, while the leisure parameter and risk aversion decrease in value. Most notably, the estimate for the loss-framing parameter increases to a value of 0.0305. The overall fit of the model moments (now 14 in total) relative to the empirical counterpart, is again very good with a criterion function value of 0.0095 and a mean relative error of 2.01%. The graphical illustration shows that both the conditional stock and the work hours shares nearly coincide with the empirical counterparts. Moreover, all moments of the conditional stock share and all but two moments of the work hours share lie within the 95% confidence bounds. Yet due to the still low values for the time discount factor and the EIS, the model-predicted financial wealth-to-income ratios are considerably lower than the ones in the data, for all age groups.

--- Figure 1 here ---

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[15] We do not display confidence bands in the figures given that the model and data lines are for some target variables very close to each other and the corresponding confidence band is narrow.

[16] This is equivalent to a loss-aversion parameter λ of 3.2048 given a narrow framing parameter b_2 of 0.1 as in the two-parameter approach of Barberis and Huang (2009). Details can be found in Appendix A.

[17] As pointed out by Fagereng et al. (2017), the very large sample size as well as the small standard errors of the targeted empirical moments explain the small standard errors of the optimal preference parameters.

The third estimation combination targets the age profiles of all three empirical variables for stockholders; optimal parameter estimates appear in column (3) of Table 3. In addition, we also report in Table 4 numerical values for the 21 moments generated by the model and the data Compared to our previous results, the estimated risk aversion, leisure, and bequest motive parameters are rather similar, while the loss-framing parameter increases to an estimate of 0.0253. The discount factor also increases substantially to a value of 0.9484, which is close to typically values used in the life-cycle literature (Gomes and Michaelides 2005; Love 2010; Ameriks et al. 2011). The EIS now grows to 1.4764, more in line with the values used in the long-run risk literature focused on explaining asset pricing patterns. In that work, the EIS is usually assumed to be above one, as theoretically shown by Bansal and Yaron (2004); the empirical evidence supports this value (Bansal et al. 2007). Again, the loss-framing parameter Λ is well above zero, underscoring the conclusion that it is pivotal in helping us match the empirical moments of our chosen target variables.

--- Table 4 here ---

The overall fit of the model is very good, with a criterion function value of 0.0540; the model matches most of the 21 moments nearly perfectly. Specifically, Panel c.1 of Figure 3 shows almost no visible difference between the moments predicted by the model and the data for the conditional stock share. The numerical values depicted in Table 4 show that the greatest absolute difference between data and model prediction is only 1.02 percentage points for the fifth age group. The fit for the work hours share is also very good; only for the first and last age group does the model predict slightly higher values than the data. Notwithstanding that point, the resulting absolute differences of 0.0154 weekly work hours for the first age group and 0.0305 for the oldest age group are still low. A similar picture arises for the financial wealth-to-income ratios. The numerical values of model and data moments are again very close to each other. The only model-predicted moment that is clearly too low compared to the empirical target is the financial wealth-to-income ratio for the oldest group (8.0885 versus 9.5641).[18] Yet the overall fit of the model is still very good, as the mean relative error for the 21 moments relative to the empirical counterparts is very low (3.60%).

In summary, it can be concluded that the life-cycle model with the proposed preference specification nicely fits the assigned empirical target variable moments, though it does less well

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[18] These results do not stem from the construction of the age groups as we also perform an analysis with 14 age groups for each variable, which is equal to the maximum number of age groups if each target variable receives equal weighting in the estimation. The fit of the estimated model shows a good fit for these 42 moments, too.

fitting the financial wealth-to-income ratio for the older groups. Our parameter estimates are mainly in line with values reported in other studies. Additionally, the estimates for both the loss-framing parameter and the bequest parameter are always well above zero, indicating that they are key parameters needed to match these empirical variables.

4.2   Comparison to Preference Specifications without Loss Framing

It is naturally of interest to compare the previous results to those of alternative preference specifications without loss framing. Accordingly, we repeat the SMM estimation for the case with three target variables over all 21 moments, but this time we use traditional CRRA- and Epstein-Zin preferences defined over consumption, leisure, and a bequest motive. Disutility of losses from risky stock investments is consequently not part of the preference specification.[19]

Estimation results for the optimal set of preference parameters appear in Panel B of Table 5. For comparison purposes, column (1) repeats the results for the model with loss framing, while columns (2) and (3) document the optimal parameter values for Epstein-Zin and CRRA preferences. The corresponding Figure 2 contrasts the age-profile of the model-implied moments with the empirical counterparts.

--- Table 5 here ---

--- Figure 2 here ---

For Epstein-Zin preferences, the parameter estimates for the leisure parameter, the EIS, and the risk aversion do not differ greatly, compared to the loss-framing case. Yet the estimates for the time discount factor are significantly lower, while the parameter for the bequest strength is much higher. Altogether, the fit of the model is considerably worse than that of the baseline model (with a criterion function value of 1.2832). The mean relative error of 16.71 here is more than four times higher than that of the model with a loss-framing component. An examination of the conditional stock share (dotted line in Figure 2) yields a good fit of the empirical moments (solid line) for the first three age groups. For older age groups, however, the Epstein-Zin model predicts a much too high or too low conditional stock share, versus the data.

Overall, then, we conclude that the explanatory power of the Epstein-Zin model is inferior to that of the loss-framing model. This is also evident for the matching results of the financial wealth-to-income ratio, where results are reasonable only for the second to fourth age groups. In contrast, for the youngest group, the ratio is too high, and for the oldest three groups, the

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[19] Given that the specifications differ with respect to the number of parameters, we reduce the length of the Sobol sequence of the parameter grid in the first step of the SMM estimation.

ratio is far too low compared to the data. Interestingly, this is the case even though the bequest parameter *b* has a significantly higher estimate (5.98 versus 3.34) than in the baseline model with loss framing. This, in turn, strengthens our conjecture that the stock market loss-framing parameter offers an important channel that the other parameters cannot achieve (even in conjunction). The work hours share for all age groups is well below both the data and that implied by the baseline model.

For the model with CRRA preferences and a bequest motive, the preference parameter estimates are similar to those for Epstein-Zin preferences. While the discount factor is even lower and close to the parameter bound, the bequest parameter is higher. The overall fit of the model is worse (the criterion function value is 1.7391) compared to that of the baseline model, and also compared to Epstein-Zin preferences. The mean relative error of 20.71% is six times higher compared to the model with loss framing. Also a direct comparison of the moments predicted in Figure 2 reveals that the explanatory power of the life-cycle model with CRRA preferences (dashed-dotted lines) does not stand up to that of the model with loss framing. The work hours still has a decent fit for all but the youngest group, is closer to the data than that of Epstein-Zin preferences and fits only slightly worse than the baseline model. Yet the conditional stock share is always above the empirical moments and the loss-framing model. In addition, the fit of the financial wealth-to-income ratio is poor, as the household accumulates more financial wealth early on in life but not enough in later ages.

4.3   Sensitivity Analysis of the Loss-Framing Parameter

Thus far, all analyses used the optimal parameter values from the SMM estimation. In this section, we perform additional sensitivity analyses on the stock market loss-framing parameter Λ, which we have shown to be central to the successful explanation of the data, especially regarding the portfolio choice decision. In particular, we are interested in the influence of the loss-framing parameter on the conditional stock share and the household stock market participation rate.

To this end, we start with the optimal set of preference parameters resulting from the structural estimation of the life cycle model with loss framing for the case with three target variables over seven age groups (Table 5 column 1). Next, the estimated optimal parameter values for α, β, γ, ψ, and b are retained, and the life-cycle model is solved repeatedly for varying parameters of the loss-framing parameter Λ.

For the analysis, we construct an equally spaced linear grid of 1,000 parameter values for Λ ranging from 0.00 to 0.05, where the latter is roughly twice the value of the respective estimate from the structural estimation. The resulting outcomes of interest generated by the various life-cycle models, namely the ratio of households participating in the stock market (left side) and the share of financial wealth participants invested in equities (right side), are displayed as contour plots in Figure 3.

--- Figure 3 here ---

For the stock market participation rate, the results show a clear pattern: the participation rate is around 1.0 for nearly the entire work phase of the household's life cycle (dark red surface) for most values of the loss-framing parameters. Moreover, for ages 60 onward, the participation rate is 1.0 for all parameter values. This is probably a result of the strong bequest motive needed to match the wealth accumulation, not counterbalanced by a stronger loss-framing motive. A loss-framing value of $\Lambda = 0.035$ is a cutoff point, after which the participation probability drops below 100%. The loss-framing parameter of $\Lambda = 0.0256$ found in the SMM estimation procedure for stockholders is clearly below this cutoff point, and it generates a model-implied participation rate for all ages of 100%, exactly as in the data. This further improves the already excellent matching result of the model with loss-framing preferences for the stockholder subgroup.

Next, we ask whether the model can replicate empirical participation rates in the entire population. Here the results are less clear: the transitions are very rapid from very high to very low participation rates of nearly zero (dark blue area). Only for a small corridor of parameter values (green to yellow area) is the participation rate consistent with empirical rates seen in the overall population (around 0.60; see also Table 2). Yet the required parameter values that ensure consistency of model predictions with empirical participation rates are not constant, but instead rise with age. Furthermore, the results are very sensitive, i.e. small deviations from the required parameter values lead to either full or no stock market participation. Both are unfavorable properties for a structural estimation of the model which also includes empirical participation rates.

The right side of Figure 3 shows that the life-cycle pattern of the conditional stock share also depends on the loss framing parameter. For low values, the share invested in stocks is decreasing with age, while it rises with age for relative high parameter values. Interestingly, for mid-level loss framing values (from 0.02 to 0.03), the stock share remains nearly constant with

age. Within this relative broad range, a parameter value of $\Lambda = 0.0256$ leads to stock shares between 0.40 - 0.45 for the different age groups that are consistent with those observed in the data. In contrast to the participation rates, small deviations from this parameter value produce only moderate changes in the model's predicted stock rates. Yet a value for the loss-framing parameter of about 0.025 leads to participation rates of 1.0 as the left side of Figure 3 demonstrates, which is perfectly appropriate for the subgroup of stockholders but not for the entire population. Conversely, the required value of at least 0.035 to bring the participation rates predicted by the model in line with the empirical values of the overall population leads to very low equity ratios of about 10% (see dark blue area right side of Figure 6), which is clearly inconsistent with the data.

Summing up, we find that both the conditional stock share and stock market participation rates are plausibly sensitive to the loss-framing parameter. Higher loss-parameter values lead to lower conditional stock shares, as well as lower participation rates. Yet the ability of the loss-framing parameter to fit model results to empirical values depends on one's objective. That is, empirical conditional stock shares are replicated very well by an appropriate choice of the loss-framing parameter: the optimal loss framing value leads to a 100% participation rates in the stock market, exactly appropriate to the subgroup of stockholders. Yet those parameter values are not suitable to explain the empirical age profile of participation rates. Of course, one must also bear in mind that the other preference parameters whose values are fixed during the analysis also affect the results, most notably those implying a strong savings motive. Next, we turn to an examination of whether a plausible parameter vector that resolves this dilemma can be achieved by varying the other preference parameters in the same way.

5   Analysis for the Entire Population

In the following section, we structurally estimate the life-cycle model with loss framing preferences as in equation (1), but now we use data for the entire population and not just stockholders as in the preceding section. Consequently, our main challenge is to explain the limited stock market participation of U.S. households. There are now four empirical target variables over the seven age groups (i.e. 28 moments), as the stock market participation rate is added. Here we define this as an indicator variable depending on whether the household has a positive allocation of financial wealth to stocks. Additionally, wealth accumulation is now proxied by the financial wealth-to-income ratio unconditional on the asset allocation. We

structurally estimate the second-stage parameters for each model again using the SMM procedure outlined in section 3.2.2.

5.1 Models

As pointed out by Gomes (2020) and Gomes et al. (2021), there are four broad explanatory approaches for stock market non-participation: preferences that exhibit first-order risk aversion, exogenous background risks, stock market participation costs, and peer effects. Our life-cycle model is capable of straightforwardly accommodating the first three approaches, which we do using three different model specifications; our goal is to evaluate whether these can explain the 28 empirical target moments.

The first approach is the baseline loss-framing model developed in the previous section, as our preference specification exhibits first-order risk aversion. The second specification introduces a heightened risk for the household; given that the focus of this study is stock market loss framing, we let the risk affect the stock market as it is the sole risk that affects both the work and the retirement phase.[20] In particular, we let the stock follow a Markov regime-switching process, in which the expected stock return shifts between two regimes with differing mean and volatility.[21] Using total return data from the S&P 500 index, the estimation utilizing the expectation-maximization (EM) algorithm results in a first regime that resembles a bear market with negative expected returns and heightened volatility levels, and a second regime resembles a bull market with positive returns and modest volatility (see Appendix B.6 for details).

Our third specification extends the baseline model by assuming pecuniary per-period stock market participation costs, as in Fagereng et al. (2017). In this case, the number of parameters in the structural estimate increases to seven because per-period participation costs are included in addition to the six preference parameters.

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[20] An alternative source of risk possible in our model framework would be a combination of labor and health risks. Labor risk could be either personal disaster risk like unemployment spells (Bagliano et al. 2019; Bagliano et al. 2021) or countercyclical labor income risk (Catherine 2022; Shen 2021) that affects the household's working life, while the health risk (Capatina 2015; Yogo 2016) would affect the retirement phase of the household.

[21] Regime-switching models have the appeal that they can generate rather flexible return distributions, including skewness, excess kurtosis, and volatility clustering (Turner et al. 1989; Pagan and Schwert 1990), while making the model still computationally tractable. Additionally, regime switching seems to make sense from a behavioral point of view, as there is evidence that individual investors see the stock market as a Markov chain. While some investors believe that past stock returns tend to persist in the future (De Bondt 1993; Greenwood and Shleifer 2014), other investors believe that stock returns tend to reverse (Dominitz and Manski 2011; Heiss et al. 2019).

As we shall show, each model specification is appealing given the evidence documented by Choi and Robertson (2020) and Bender et al. (2022) that loss aversion, disaster risk, and fixed participation costs are all important factors in households' portfolio choice decisions.

5.2 Estimation Results

The estimation results are presented in Table 6, while the model-implied moments are compared to the empirical moments in Figure 4.

--- Table 6 here ---

--- Figure 4 here ---

Model 1 (Loss Framing) has similar parameter estimates as the most complex model for stockholders. Only the loss-framing parameter rises substantially in value to an estimated 0.0356. The overall fit of the model is not good, with a criterion function value of 2.999 and an MRE of 0.2420. The worse fit stems mainly from the stock market participation rate, which is increasing in age and predicts full participation for the oldest three age groups. The fit for the other three variables is decent, although the conditional stock share for the younger age groups is too low, and too high for the oldest age groups. The financial wealth-to-income ratio is also too low for the oldest groups. The work hours share is flat at a value of around 0.4. These results support the conjecture that a higher loss-framing parameter decreases stock market participation, but it also simultaneously pushes the conditional stock share down, making a simultaneous fit for these variables challenging. Overall, this confirms our earlier point that the loss-framing preference specification alone does not discourage households from completely shying away from the stock market

Model 2 (Loss Framing with Regime Switching) extends Model 1 by assuming that the stock follows a two-dimensional regime-switching process. It has similar parameter estimates as the model for stockholders and Model 1 except for the loss-framing parameter. The stock market loss-framing parameter (estimated at 0.0860) is more than double, and three times the value found in models without regime-switching. This is a result of the regime-switching behavior of the stock since the stock process has different parameters for each regime which themselves enter the loss-framing part of the preferences via both the conditional expectation of losses as well as the probability of losses. For the bear market regime 1, which has a negative drift and high volatility in the stock process, the implied loss aversion skyrockets. Consequently, the model set-up implies countercyclical loss aversion. This behavior is in line with evidence reported by Hwang and Satchell (2010) that loss aversion changes with market

conditions. The overall fit of the model is reasonable with a criterion function value of 1.7002 and an MRE of 0.1884; both are lower than the values for the one-regime model. The better fit mainly stems from the stock market participation rate, which nevertheless is always higher than in the data. For the oldest four age groups, the stock market participation rate stays constant at 0.85, equal to the invariant probability of the normal regime in the switching process. The stress regime is so disastrous that the affected household exits the stock market. The conditional stock share is again increasing in age; at first it remains below its empirical counterpart, and then it rises above it. The financial wealth-to-income is increasing in age and is a bit too high for the last two age groups. The work hours share does not change compared to the previous model, a consequence of choosing a work hour share of 0.40 as threshold for receiving an overtime wage premium. Hence, in most simulation paths, the household will seek to take advantage of higher wages, but this is moderated by its strong preference for leisure.

Model 3 (Loss Framing with Participations Costs), which extends Model 1 by including stock market participation costs, has different parameter estimates compared to the previous two. Most notably, the risk aversion decreases to a value of 2.8991, while the bequest parameter is quite high, at 5.2232. The loss-framing parameter decreases to a value of 0.0217, more in line with the estimates for the stockholder model. The estimated value for χ of 1.9012, defined in units of permanent income, translates into expected annual participation costs ϕ of 720.14 dollars for a 25 year old household. This is a higher value than typically found in the literature (see Fagereng et al. 2017). The overall fit of the model is very good with a criterion function value of 0.4308 and an MRE of 0.0956. The participation rate and the conditional stock share show a hump-shape as observed in the data, while the financial wealth-to-income fits similarly as in the model without participation costs.

The preceding analysis illustrates that, in order to match the population moments for all four target variables, the proposed life-cycle model with stock market loss framing needs to be coupled with per-period stock market participation costs. The other model specifications have difficulty matching observed stock market participation patterns.

6   Analysis with Heterogeneous Agents

Our structurally estimated model with a single representative agent matches the empirical target variables for the overall population very well. Yet the model's explanatory power as measured by the MRE is still lower compared to the model for the stockholders alone (9.6% versus 3.6%). Moreover, the model is unable to account for the empirical difference in the age

patterns of financial wealth-to-income ratio for households who do not participate in the stock market, versus the stockholders alone. For example, Table 2 shows that stockholders have about six times as much financial wealth in the retirement phase as non-stockholders. To tackle both issues, we perform an analysis with ex-ante heterogeneous agents.

6.1 Approach

There is general consensus that individual investors exhibit pronounced heterogeneity in preferences (Ghosh et al. 2020; Calvet et al. 2021) and beliefs (Giglio et al. 2021; Meeuwis et al. 2022).[22] In our approach, all heterogeneity is preference-based and not belief-based. Moreover, we differentiate ex ante between three groups of investors defined as follows: the first group represents stockholders facing no participation costs; the second represents non-stockholders who never participate in the stock market for reasons other than preferences and participation costs; and the third group, which we refer to as the blended group, consists of households that do face some stock market participation costs. The rationale behind this approach is that participation costs cannot be the sole explanation for stock market non-participation, as even newly wealthy households do not always participate in the stock market (Andersen and Nielsen 2011; Briggs et al. 2021).[23]

The weights for the different groups are determined ex ante, and are assumed to be 30% stockholders, 20% non-stockholders, and 50% for the blended group. The first two weights are inferred from the SCF data from 1989 to 2019. In the dataset, 24.73% of the households held stocks solely in quasi-liquid retirement accounts, which corresponds to facing zero stock market participation costs since these accounts are usually set up by the employer and nowadays predominantly default workers to a target-date fund with a substantial equity portion (Mitchell and Utkus 2021; Parker et al. 2021).[24] The remaining households we assume to pay no participation costs for other possible reasons such as having bequeathed equity wealth. The assumed weight of 20% for the non-stockholder group represents roughly half of the 38.20% households not participating in the stock market reported in the SCF. The weight of the blended

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[22] There is also substantial heterogeneity in returns of wealth as documented by Campbell et al. (2019), Bach et al. (2020), and Fagereng et al. (2020).

[23] In addition, the non-stockholder group can reflect other possible individual-level explanations outside the scope of our modeling framework, such as financial literacy (Van Rooij et al. 2011), trust (Guiso et al. 2008), and peer effects (Hong et al. 2004); such information is not available in our dataset.

[24] This is in effect since the Pension Protection Act (PPA) of 2006.

group, which consists of both stockholders and non-stockholders, is a residual weight such that the total weight of the groups sums to one.

Contrary to other approaches in the literature that aggregate each group to jointly match population moments, we choose to structurally estimate the household group for stockholders and non-stockholders in order to match their standalone target variables, while the blended group is estimated such that it minimizes the residual distance that is needed for the three-group model to match the population moments.

6.2 Estimation Results

The optimal parameter values for the three distinct household groups are displayed in Panel B of Table 7. Results for the first household group are equivalent to those having three target variables for stockholders in Section 4, so we do not cover them again. The model for non-stockholders must only fit the age pattern of the financial wealth-to-income ratio and the work hours share. Given that this type of household does not have the opportunity to invest in the stock market, it does not exhibit loss framing. The resulting estimates yield a low discount factor, as well as very low risk aversion and a very high bequest motive. The rather low risk aversion of about 1.5 is consistent with prior studies on structural life cycle models for U.S households which do not consider portfolio choice with risky assets. For example, Gourinchas and Parker (2002) estimate values around 1.5 and De Nardi et al. (2010) of 3.5. In asset pricing studies and in structural models with risky investments (as ours), much higher values are routinely used. The overall fit of the model is very good with a criterion function value of 0.0229 and a mean relative error of 0.0316. The blended group has similar parameter estimates as the population model with participation costs shown previously in Table 6 (column 3). Most notably, the estimated loss-framing parameter takes the value of 0.0088, significantly lower than the corresponding estimate of 0.0217 for the population. The estimate of χ increases from 1.9012 to a value of 2.4899, which translates into expected annual participation costs ϕ of 943.13 dollars for a 25 year old household. Both results are not unexpected, as the representative agent model with participation costs is similar to a combination of all three groups in the heterogeneous agent model. Since the group of shareholders is assumed to participate fully in the stock market and has a higher weight in the total population than the group of non-shareholders, participation costs for the mixed group act as the main driving force to control stock market participation rates for the total population.

The same logic applies to the loss framing parameter as the main driver to control the conditional stock ratio in the total population. This is higher for the shareholder group than for the model with only one representative agent (0.0253 versus 0.0217). To balance this out, the degree of loss framing is lower for the blended group (0.0088).

When it comes to the separate fit of the blended model, both the criterion function value of 0.2957 and the mean relative error of 0.0721 illustrate the very good model fit. The goodness-of-fit metrics for the combined three-group model appear in Panel C of Table 7, while the model-implied moments of the combined three-group model can be contrasted to the empirical moments in Figure 5 and Table 8.[25] The model is able to match all four assigned variables very well. The life cycle profiles of the participation rate and the conditional stock share are hump-shaped, as in the data. Moreover, the relative difference between model and data moments is always below 10%. The financial wealth-to-income ratios implied by the model are now strictly increasing in age and are always close to the empirical counterpart. The work hours share is stable around a value of 0.405 for most age groups, and hence a bit below the share in the data. The model's overall excellent fit is numerically confirmed by a criterion function value of 0.0796 and a mean relative error of 0.0438, close to the numbers of the standalone models for stock- and non-stockholders.

Overall, this analysis yields two conclusions. First, our life-cycle model with preferences that include stock market loss framing fits the data moments for different investor groups of the population very well. Second, our heterogeneous agent model can account for the observed life-cycle patterns of portfolio choice, wealth accumulation, and labor supply of U.S. households.

6.3 Comparison of Estimated Models for Non-Targeted Empirical Phenomena

In this section, we analyze the implications of our models for other empirical phenomena in household finance. Inasmuch as investment decisions are the primary objective of this study, the analyses to follow focus on differences in the life-cycle wealth accumulation between stock- and non-stockholders, and household portfolio choice decisions for different levels of financial wealth irrespective of age. Results for both evaluations appear in Figure 6. We only compare

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[25] The predicted moments for each of the three subgroups are contrasted to the data moments both, numerically and graphically in Appendix E.

results from the single-representative-agent model with stock market participation costs (dashed lines) with those from the heterogeneous agent model (dotted lines), given that both are far superior to the other model specifications in explaining the population moments (solid lines). We discuss these results in the following two sections.

--- Figure 6 here ---

6.3.1 Wealth Accumulation by Investor Type

As illustrated in the preceding section, the observed financial wealth-to-income ratios for stockholders at different ages are three to six times higher those of non-stockholders. This constitutes an important empirical fact that life-cycle models should account for, if they aim not only to explain the investment decisions for the population as a whole, but also seek to explain investment decisions of population subgroups. To examine this point, we examine both the data and the model results for stockholders versus non-stockholders. Figure 6 (Panel a) displays the financial wealth-to-income ratios for both types of households.

For the stockholder group (Figure 6a.1), both life-cycle models produce an increasing ratio of financial wealth to income over time, but below the observed values (solid line). Yet the ratio for the heterogeneous agent model (dotted line) is much higher and closer to the data, compared to that of the single-agent model (dashed line). This is numerically reflected in an MRE of 0.2217 vs. an MRE of 0.3625. The same tendency is observed for the non-stockholder group (Figure 6.a.2), where the model-implied moments are above the empirical ones. Again, the heterogeneous agent model gives a better fit for that group, with an MRE of 0.5708 against an MRE of 1.5841 for the singe-agent model.

Although not the overall objective of the structural estimation, the estimated heterogeneous agent model is evidently able to produce a significantly higher difference in the financial wealth-to-income ratios, compared to the single agent model.

6.3.2 Portfolio Choice by Financial Wealth

So far, our focus has been on life-cycle and thus age effects. Next, we focus on wealth effects and investigate whether the models for the population can account for the reality that the stock market participation and stock share increase with financial wealth in the U.S. (Wachter and Yogo 2010; Favilukis 2013; Kuhn et al. 2020).

To this end, we start with the same sample of households used to obtain the empirical targets for the structural estimation; this group is derived from SCF waves 1989 to 2019, and it includes households age 25 to 80. Similar to Wachter and Yogo (2010), we first sort all

observations into deciles of financial wealth within each wave and then taking the mean of the stock participation rate and the conditional stock share within each decile over all waves. An analogous procedure is used for the two life-cycle models by sorting the simulated households age 25 to 80 into deciles of financial wealth. The two graphs below in Figure 6 (Panel b) displays the results.

The empirical evidence shows that the stock market participation rate is steadily increasing in financial wealth, from a level below 0.1 for the first decile, to a value near 1.0 for the top decile. Moreover, the curve is concave. The participation rate in our life-cycle models is also increasing in wealth, but while the rate for the singe-agent model flattens out for all but the first decile at around 0.6, the rate implied by the heterogeneous agent model shows first higher then lower rates than in the data, but reaches nearly full participation for the highest wealth decile. The less than perfect fit for the first three deciles is reflected in the overall MRE of 0.5519 for the stock market participation rate, though this is still substantially lower than the MRE of 0.9137 for the single-agent model.

The empirical pattern of the conditional stock share is increasing in wealth, from a level of 0.4 for the first decile, to nearly 0.55 for the top decile. The life-cycle simulations match the empirical conditional stock share for most deciles in the population rather well, although the share is hump-shaped in wealth. This result is expected, as both groups that target the conditional stock share show a hump-shaped share and increasing wealth by age, which is why the conditional stock share by wealth will be similar to the conditional stock share by age. The good fit for both models is reflected in the overall MRE of 0.0815 for the heterogeneous agent model and an MRE of 0.1090 for the single-agent model. Thus, to a large extent, the heterogeneous agent model generates the same pattern and level for portfolios choice by financial wealth as in the data (although they are not explicitly targeted). Given that the single-agent model is also able to produce an increasing conditional stock share in financial wealth, our model provides an alternative preference-based explanation to that offered by Wachter and Yogo (2010), who use a life-cycle model with a non-homothetic preference specification to explain the increasing conditional stock share in wealth.[26]

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[26] The best-fitting model for a stockholder and the population model with regime switching also give a rising conditional stock share in financial wealth, indicating that the proposed preferences with stock market loss framing are a driving force behind this result.

7 Conclusions

This paper structurally estimates the parameters of a parsimonious life-cycle model of optimal consumption, portfolio choice, and labor supply that incorporates stock market loss-framing in the specification of preferences. Stock market loss-framing is defined as the additional disutility that a household experiences by the expected shortfall on stock investment return below a benchmark return. The model with stock market loss-framing can account for the empirical age-patterns of the stock share, financial wealth-to-income ratios, and work hours of U.S. stockholders. This is achieved with plausible preference parameter estimates, while the stock market loss-framing parameter and the bequest parameter are always well above zero. The predictive power of the model incorporating stock market loss-framing is also far superior to that of models with preference specifications that lack stock market loss-framing.

Extending the model by adding in age-dependent per-period stock market participation costs simultaneously explains the empirical age-profiles of stock market participation, portfolio choice, wealth accumulation, and the labor supply patterns for the entire population. Finally, a model with heterogeneous agents further improves explanatory power, and it generates the observed discrepancy in wealth accumulation between stockholders and non-stockholders. Moreover, such a life-cycle model with stock market loss-framing, per-period participation costs, and heterogeneous agents explains the rise in stock market participation and conditional stock share with financial wealth.

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Table 1: First-Stage Model Parameter Values

This table presents the values for all first-stage model parameters. A $\times$ in the last column indicates that the respective parameter is estimated. Details on data and estimation methods for those parameters appear in Appendix B. All other parameters are calibrated. $\widetilde{age}$ denotes the integer age of the household (equal to $t + 24$) divided by 100 and GPD is the acronym for the generalized Pareto distribution with its three parameters.

Description	Symbol	Value/Source	Est.
Household Characteristics			
Starting Age	$t_0 + 24$	25	
Ending Age	$T + 24$	100	
Survival Probabilities	p_t	HMD period life table 2017	$\times$
Capital Market			
Drift Stock Return	μ	0.0637	$\times$
Volatility Stock Return Shock	σ_η	0.1564	$\times$
Gross Risk-Free Rate	R_f	1.0077	$\times$
Capital Gains Tax Rate	t_c	0.15	
Management Fees	κ	0.0018	
Scaling Factor Stock Market Part. Costs	F_t	$0.0152(t + 24)$	$\times$
Labor Income			
Age-Independent Component Wage Rate	$w(\cdot)$	1.6428	$\times$
Age-Dependent Component 1 Wage Rate	$w(\cdot)$	$6.4680(\widetilde{age})$	$\times$
Age-Dependent Component 2 Wage Rate	$w(\cdot)$	$-9.2595(\widetilde{age})^2$	$\times$
Age-Dependent Component 3 Wage Rate	$w(\cdot)$	$3.3715(\widetilde{age})^3$	$\times$
Under-Time Discount Wage Rate	$w(\cdot)$	-0.1979	$\times$
Over-Time Premium Wage Rate	$w(\cdot)$	0.2126	$\times$
Volatility Permanent Wage Shock	σ_ϑ	0.0896	$\times$
Volatility Transitory Wage Shock	σ_φ	0.2463	$\times$
Correlation Wage and Stock Shock	$\rho_{\log(\xi),\eta}$	0.0000	$\times$
Upper Bound Labor Share	$\widetilde{N}$	0.6	
Lower Bound Labor Share	$\underline{N}$	0.0	
Yearly Waking Hours	Υ	5200	
Labor Income Tax Rate	t_l	0.3	
Retirement Income			
Benefit Replacement Ratio	ζ	0.5838	$\times$
Proportional Medical Expenditures	λ	0.1175	$\times$
Volatility Net Retirement Income Shock	σ_ε	0.3006	$\times$
Average Lifetime Working Hours	$\overline{N}$	0.4	
Retirement Income Tax Rate	t_r	0.1	
Retirement Age	$K + 25$	67	
Housing Costs			
Age-Independent Component Cost Function	$h(\cdot)$	0.5212	$\times$
Age-Dependent Component 1 Cost Function	$h(\cdot)$	$-1.5218(\widetilde{age})$	$\times$
Age-Dependent Component 2 Cost Function	$h(\cdot)$	$2.6889(\widetilde{age})^2$	$\times$
Age-Dependent Component 3 Cost Function	$h(\cdot)$	$-1.7591(\widetilde{age})^3$	$\times$
Distribution Starting Financial Wealth Ratio			
Stockholders	$(W/Y)^{init.}$	$GPD(0.0000, 0.0751, 1.0754)$	$\times$
Population	$(W/Y)^{init.}$	$GPD(0.0000, 0.1156, 1.0618)$	$\times$

Table 2: Estimation Results for Empirical Target Variables

This table presents the results of the estimation of the six empirical variables (mean values grouped into seven equidistant age buckets) which are targeted in the structural estimation throughout the paper. For each target variable, age, time, and cohort effects are disentangled using the method of Deaton and Paxson (1994). The stock participation rate, the conditional stock share, and the financial wealth-to-income ratios rely on the SCF (waves 1989 – 2019) where only observations with a positive dollar amount of financial wealth and a positive dollar amount of labor income are considered. The work hours share relies on the PSID (waves 1975 – 2017) where only observations of employed individuals with an hourly wage rate above $5 or below the 99th percentile of each wave are considered. Stock market participation is defined as an indicator variable equal to 1 if the total dollar stock allocation is above zero (column 2). The conditional stock share is the fraction of total equity to total financial wealth given a non-zero allocation to stocks (column 3). The financial wealth-to-income ratio for stockholders is defined as the ratio of total financial wealth to annual labor or retirement income, given: (i) a positive allocation to stocks for the stockholder group (column 4), (ii) a zero allocation to stocks the for non-stockholders group (column 5), and (iii) for the entire population (column 6). The work hours share is defined as the weekly work hours reported in PSID divided by an assumed 100 waking hours per week (column 7). Each of the seven age groups for the stock participation rate, the conditional stock share, and financial wealth-to-income ratios, consist of eight consecutive years of life, starting from 25 and ending with 80. Each age group for the work hours share uses six consecutive years of life, starting from 25 and ending with 66. Standard errors for each regression coefficient are displayed in parentheses below each, obtained by bootstrapping the data using 1,000 resamples. One, two, and three asterisks indicate that the coefficient is statistically significant at the 10%, 5%, and 1% significance level, respectively.

			Empirical Target Variable			
	Stock Part. Rate	Conditional Stock Share	Financial Wealth-to-Income Ratio			Work Hours Share
			Stockholder	Non-Stockholder	Population	
Age Group 1	0.5203***	0.3870***	0.7471***	0.2626***	0.5245***	0.4184***
	(0.0105)	(0.0078)	(0.0762)	(0.0516)	(0.0530)	(0.0049)
Age Group 2	0.5837***	0.4240***	1.0026***	0.3511***	0.6890***	0.4190***
	(0.0118)	(0.0086)	(0.0908)	(0.0682)	(0.0570)	(0.0049)
Age Group 3	0.6427***	0.4401***	1.6499***	0.5517***	1.1201***	0.4189***
	(0.0132)	(0.0090)	(0.1306)	(0.0983)	(0.0743)	(0.0048)
Age Group 4	0.6477***	0.4632***	2.6184***	0.7301***	1.7355***	0.4210***
	(0.0120)	(0.0098)	(0.1823)	(0.1340)	(0.0983)	(0.0050)
Age Group 5	0.6546***	0.4500***	4.7334***	0.8600***	2.9938***	0.4184***
	(0.0123)	(0.0098)	(0.2778)	(0.1966)	(0.1310)	(0.0053)
Age Group 6	0.5614***	0.4201***	7.4298***	1.4551***	4.1488***	0.4107***
	(0.0149)	(0.0108)	(0.3799)	(0.2633)	(0.1956)	(0.0051)
Age Group 7	0.5288***	0.4286***	9.5641***	1.5251***	4.8879***	0.3897***
	(0.0130)	(0.0113)	(0.4118)	(0.2961)	(0.1874)	(0.0057)
Observations	207,409	128,057	128,057	79,352	207,409	1,305,385

Table 3: Structural Estimation Results for Stockholders

This table presents structural estimation results for the model with loss framing preferences for stockholders. Panel A indicates which combination of target variable moments is used for that estimation, and Panel B presents estimation results. The standard error for each parameter appears in parentheses, obtained by bootstrapping the simulated data using 1,000 resamples. The criterion function value is the sum of the squared percentage deviations of the model and the data moments and is the targeted distance function in the estimation. The mean relative error denotes the mean of the percentage deviation of the model moments and the targeted data moments. The number of moments denotes the total number of targeted moments in the structural estimation.

	Empirical Target Variable Combination		
	(1)	(2)	(3)
Panel A: Targeted Variables			
Conditional Stock Share	$\times$	$\times$	$\times$
Financial Wealth-to-Income Ratio			$\times$
Work Hours Share		$\times$	$\times$
Panel B: Parameters			
Discount Factor (β)	0.8417	0.8982	0.9484
	(2.14e-06)	(2.44e-07)	(3.04e-07)
Leisure (α)	1.3518	1.1693	1.4758
	(4.49e-06)	(3.74e-07)	(1.53e-07)
EIS (ψ)	0.2677	0.5582	1.4818
	(5.47e-06)	(8.09e-07)	(1.62e-07)
Risk Aversion (γ)	8.6455	8.3211	9.8435
	(2.47e-06)	(1.09e-06)	(2.91e-06)
Bequest (b)	3.4424	4.0110	3.3426
	(1.69e-06)	(9.57e-07)	(3.14e-07)
Loss Framing (Λ)	0.0201	0.0305	0.0253
	(3.68e-07)	(3.99e-07)	(6.10e-08)
Criterion Function Value	0.0016	0.0095	0.0540
Mean Relative Error (MRE)	0.0132	0.0201	0.0360
Number of Moments	7	14	21

Table 4: Model vs. Data Moments in the Stockholder Model

This table compares the moments implied by the structural estimation and the empirical moments for the stockholder group; the key variables simultaneously targeted are conditional stock share, financial wealth-to-income ratio, and work hours share. Empirical moments are estimated using the SCF (1989–2019) and the PSID (1975–2017), and by utilizing the methodology of Deaton and Paxson (1994). Model moments are those implied by the parameter combination that gives the best fit in the structural estimation; these are the mean values from 100,000 simulated life cycles based on optimal feedback controls. Each of the seven age groups for the conditional stock share and financial wealth-to-income ratios, consist of eight consecutive years of life, starting from 25 and ending with 80. Each age group for the work hours share uses six consecutive years of life, starting from 25 and ending with 66.

	Age Group						
	1	2	3	4	5	6	7
Panel A: Conditional Stock Share							
Data	0.3870	0.4240	0.4401	0.4632	0.4500	0.4201	0.4286
Model	0.3909	0.4264	0.4406	0.4539	0.4398	0.4218	0.4378
Panel B: Financial Wealth-to-Income Ratio							
Data	0.7471	1.0026	1.6499	2.6184	4.7334	7.4298	9.5641
Model	0.7731	1.0298	1.5832	2.6962	4.6735	8.3233	8.0885
Panel C: Work Hours Share							
Data	0.4184	0.4190	0.4189	0.4210	0.4184	0.4107	0.3897
Model	0.4338	0.4076	0.4084	0.4117	0.4115	0.4102	0.4202

Table 5: Structural Estimation Results for Preferences without Loss Framing

This table compares structural estimates of the baseline preference specification with Loss Framing (column 1) with Epstein-Zin preferences (column 2), and CRRA preferences (column 3) for stockholders. Panel A indicates the combination of target variable moments used for the estimation and Panel B presents the estimation results. The standard error for each parameter is displayed in parentheses, obtained by bootstrapping the simulated data using 1,000 resamples. The criterion function value is the sum of the squared percentage deviations of the model and the data moments; this is the targeted distance function in the estimation. The mean relative error denotes the mean of the percentage deviation of the model moments and the targeted data moments. The number of moments denotes the total number of targeted moments in the structural estimation.

	Preference Specification		
	(1)	(2)	(3)
	Loss Framing	Epstein-Zin	CRRA
Panel A: Targeted Variables			
Conditional Stock Share	$\times$	$\times$	$\times$
Financial Wealth-to-Income Ratio	$\times$	$\times$	$\times$
Work Hours Share	$\times$	$\times$	$\times$
Panel B: Parameters			
Discount Factor (β)	0.9484	0.8626	0.8390
	(3.04e-07)	(7.36e-08)	(1.80e-07)
Leisure (α)	1.4758	1.2609	1.3092
	(1.53e-07)	(1.87e-07)	(1.93e-07)
EIS (ψ)	1.4818	1.0212	
	(1.62e-07)	(1.82e-07)	
Risk Aversion (γ)	9.8435	9.9717	8.0502
	(2.91e-06)	(8.89e-07)	(5.10e-07)
Bequest (b)	3.3426	5.9895	6.5556
	(3.14e-07)	(8.12e-07)	(4.83e-07)
Loss Framing (Λ)	0.0253		
	(6.10e-08)		
Criterion Function Value	0.0540	1.2832	1.7391
Mean Relative Error (MRE)	0.0360	0.1671	0.2071
Number of Moments	21	21	21

Table 6: Structural Estimation Results for the Population Models

This table presents the results of the structural estimation for the models for the population. The baseline preference specification (Loss Framing) appears in column 1, the loss-framing model where the stock process follows a regime-switching process (Loss Framing + Regime Switching) in column 2, and the loss-framing model with participation costs (Loss Framing + Participation Costs) in column 3. Panel A indicates the combination of target variable moments used for the estimation, and Panel B presents results. The standard error for each parameter is displayed in parentheses, obtained by bootstrapping the simulated data using 1,000 resamples. The criterion function value is the sum of the squared percentage deviations of the model moments and the data moments and is the targeted distance function in the estimation. The mean relative error denotes the mean of the percentage deviation of the model moments and the targeted data moments. The number of moments denotes the total number of targeted moments in the structural estimation.

	Model		
	(1) Loss Framing	(2) Loss Framing + Regime Switching	(3) Loss Framing + Participation Costs
Panel A: Targeted Variables			
Stock Market Participation Rate	×	×	×
Conditional Stock Share	×	×	×
Financial Wealth-to-Income Ratio	×	×	×
Work Hours Share	×	×	×
Panel B: Parameters			
Discount Factor (β)	0.9378	0.9364	0.8854
	(5.64e-08)	(1.02e-07)	(4.79e-08)
Leisure (α)	1.5934	1.5225	1.5090
	(9.27e-08)	(8.22e-08)	(1.61e-08)
EIS (ψ)	1.1905	1.2193	0.5163
	(4.57e-07)	(8.65e-08)	(6.45e-07)
Risk Aversion (γ)	8.5205	10.1594	2.8991
	(4.35e-07)	(5.02e-07)	(2.75e-07)
Bequest (b)	2.8361	3.7693	5.2232
	(4.53e-08)	(2.93e-07)	(3.13e-07)
Loss Framing (Λ)	0.0356	0.0860	0.0217
	(8.95e-08)	(4.54e-08)	(4.73e-08)
Participation Costs (χ)			1.9012
			(1.92e-07)
Criterion Function Value	2.9999	1.7002	0.4308
Mean Relative Error (MRE)	0.2420	0.1884	0.0956
Number of Moments	28	28	28

Table 7: Structural Estimation Results for the Heterogeneous Agent Model

This table presents the results of the structural estimation for the heterogeneous agent model. The three types of investors are (1) stockholders with loss framing and no participation costs (Stockholder), (2) non-stockholders with no loss framing and no participation costs (Non-Stockholder), and (3) stock- and non-stockholders with loss framing and participation costs (Blended). Panel A indicates the combination of target variable moments used in the structural estimation, and Panel B presents the parameters of the model with generates the best matching fit. The blended model is estimated such that it fits the residual distance needed for the entire three-group model to fit the stock market participation rate, the conditional stock share, the financial wealth-to-income ratio, and the work hours share for the entire population. The standard error for each parameter is displayed in parentheses, obtained by bootstrapping the simulated data using 1,000 resamples. Panel C displays the fit of the combined three-group model that consists of 30% stockholders, 20% non-stockholders, and 50% blended households. The criterion function value is the sum of the squared percentage deviations of the model moments and the data moments and is the targeted distance function in the estimation. The mean relative error denotes the mean of the percentage deviation of the targeted model moments and the data moments. The number of moments denotes the total number of targeted moments in the structural estimation.

|	Group		
	(1) Stockholder	(2) Non-Stockholder	(3) Blended
Panel A: Targeted Variables			
Stock Market Participation Rate Blended			×
Conditional Stock Share	×		×
Fin. Wealth-to-Income Ratio Stockholder	×		
Fin. Wealth-to-Income Ratio Non-Stockholder		×	
Fin. Wealth-to-Income Ratio Blended			×
Work Hours Share	×	×	×
Panel B: Parameters			
Discount Factor (β)	0.9484	0.8302	0.8523
	(3.04e-07)	(1.66e-08)	(1.43e-08)
Leisure (α)	1.4758	1.1487	1.5280
	(1.53e-07)	(2.07e-08)	(1.06e-08)
EIS (ψ)	1.4818	1.0796	0.4641
	(1.62e-07)	(3.05e-08)	(1.17e-08)
Risk Aversion (γ)	9.8435	1.4948	2.9349
	(2.91e-06)	(1.69e-08)	(2.49e-08)
Bequest (b)	3.3426	6.5964	6.0135
	(3.14e-07)	(1.52e-07)	(4.67e-08)
Loss Framing (Λ)	0.0253		0.0088
	(6.10e-08)		(1.11e-08)
Participation Costs (χ)			2.4899
			(1.37e-08)
Criterion Function Value	0.0540	0.0229	0.2957
Mean Relative Error (MRE)	0.0360	0.0316	0.0721
Number of Moments	21	14	28
Panel C: Heterogeneous Agent Model	Population = 30% · (1) + 20% · (2) + 50% · (3)		
Criterion Function Value	0.0796		
Mean Relative Error (MRE)	0.0438		
Number of Moments	28		

Table 8: Model vs. Data Moments for the Heterogeneous Agent Model

This table shows for four target variables the age-dependent empirical moments using SCF (1989–2019) and PSID (1975–2017) data vis-a-vis the moments implied by the structural estimation of the heterogeneous agent model with three investor types: 30% stockholders, 20% non-stockholders, and 50% blended households. For each subgroup we use the parameter values that gives the best fit in the structural estimation as reported in Table 7 Panel B and generate mean values from 100,000 simulated life cycle profiles based on optimal feedback controls. Subsequently the results of the subgroups are aggregated into the entire population and summarized in seven age buckets. Each of the seven age groups for the stock participation rate, the conditional stock share, and the financial wealth-to-income ratio consist of eight consecutive years of life, starting from 25 and ending with 80. Each age group for the work hours share uses six consecutive years of life, starting from 25 and ending with 66.

	Age Group						
	1	2	3	4	5	6	7
Panel A: Stock Market Participation Rate							
Data	0.5190	0.5805	0.6396	0.6447	0.6521	0.5586	0.5255
Model	0.5134	0.5928	0.6057	0.6297	0.6177	0.5585	0.5744
Panel B: Conditional Stock Share							
Data	0.3870	0.4240	0.4401	0.4632	0.4500	0.4201	0.4286
Model	0.3851	0.4188	0.4539	0.4847	0.4654	0.4380	0.3936
Panel C: Financial Wealth-to-Income Ratio							
Data	0.5264	0.6898	1.1200	1.7333	2.9880	4.1355	4.8625
Model	0.5640	0.7448	1.0747	1.6069	2.5555	4.2540	4.5502
Panel D: Work Hours Share							
Data	0.4184	0.4190	0.4189	0.4210	0.4184	0.4107	0.3897
Model	0.4088	0.4048	0.4047	0.4036	0.4048	0.4020	0.4017

# Figure 1: Matching Results: Stockholder Model for Target Variable Combinations

(a) Target Variables: Cond. Stock Share

(a.1) Conditional Stock Share (a.2) Fin. Wealth-to-Income Ratio (a.3) Work Hours Share
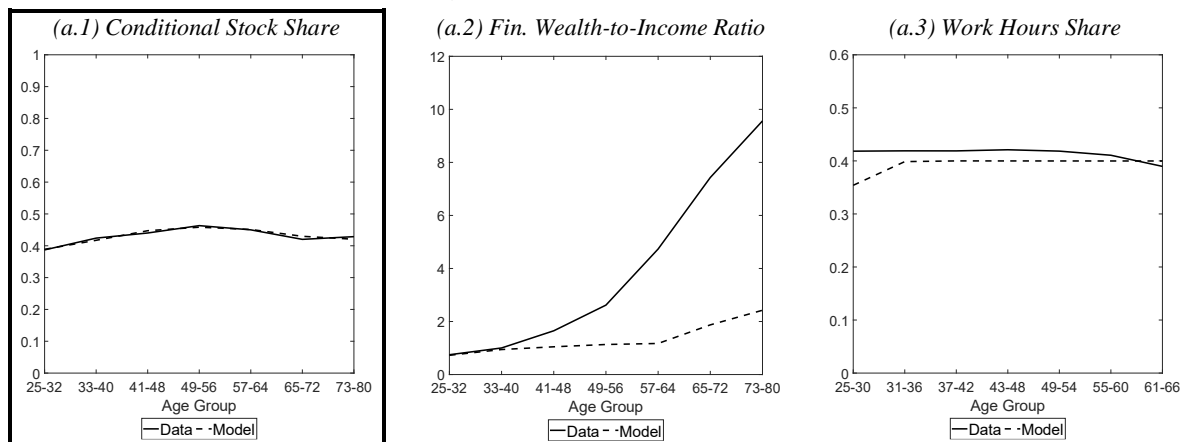
(b) Target Variables: Cond. Stock Share, Work Hours Share

(b.1) Conditional Stock Share (b.2) Fin. Wealth-to-Income Ratio (b.3) Work Hours Share

## (c) Target Variables: Cond. Stock Share, Fin. Wealth-to-Income Ratio, Work Hours Share

(c.1) Conditional Stock Share (c.2) Fin. Wealth-to-Income Ratio (c.3) Work Hours Share
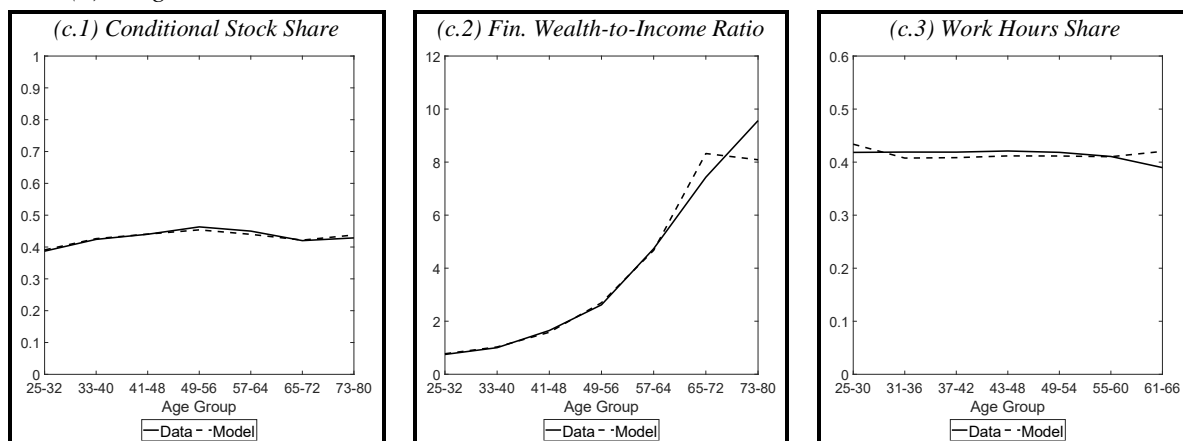
Notes: This figure compares for three target variable combinations the age-dependent empirical moments using SCF and PSID data (black line) vis-a-vis the moments implied by the structural estimation of the respective life-cycle model (dashed line), which are the mean values from 100,000 simulated life cycles scenarios based on optimal feedback controls. Each Panel with three sub-figures represents one target combination of the structural estimation procedure as outlined in Table 3 Panel A. A black edging around a subfigure indicates that the respective variable is targeted during the structural estimation.

Figure 2: Matching Results: Stockholder Model for Preferences without Loss Framing
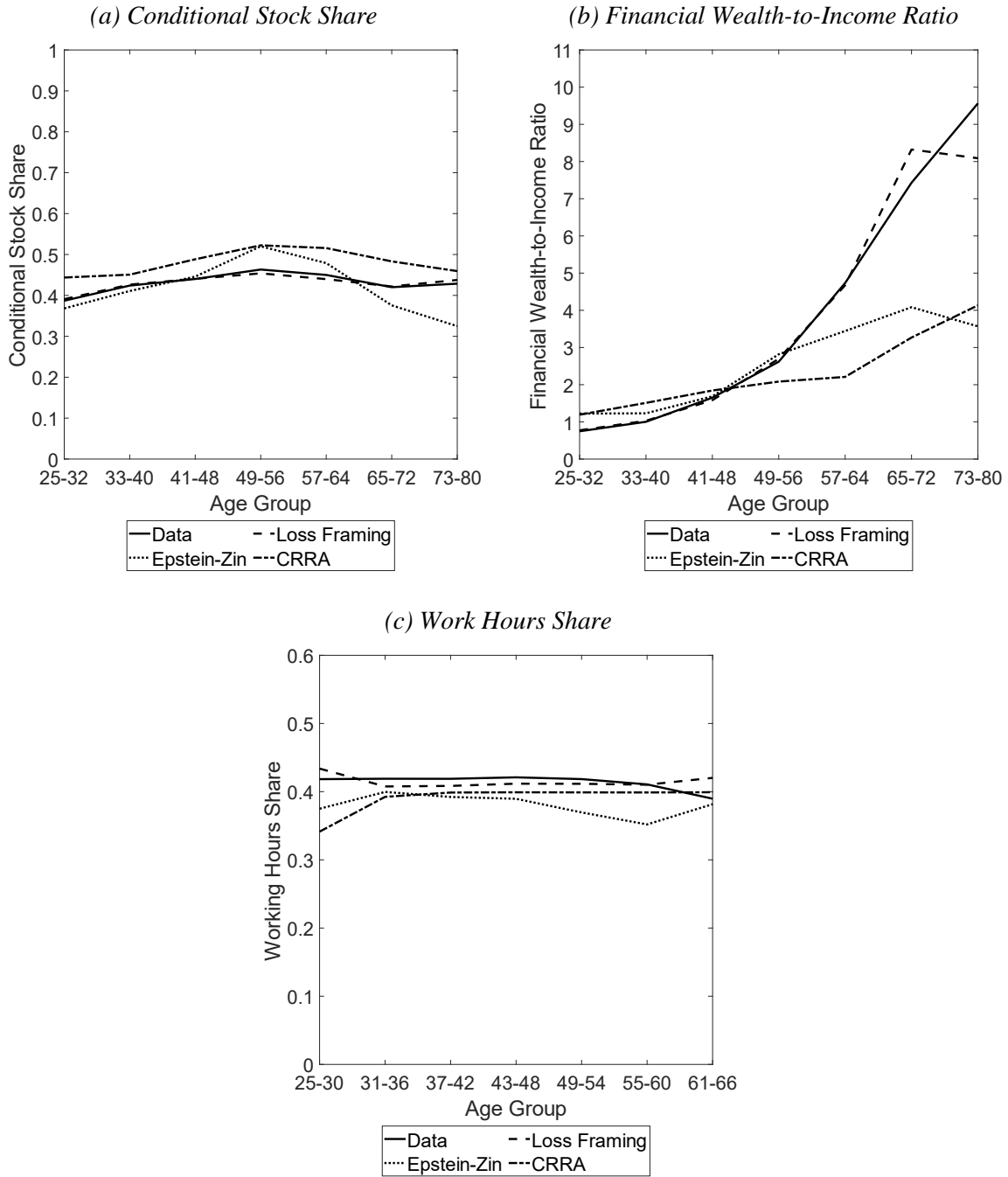
(a) Conditional Stock Share

(b) Financial Wealth-to-Income Ratio

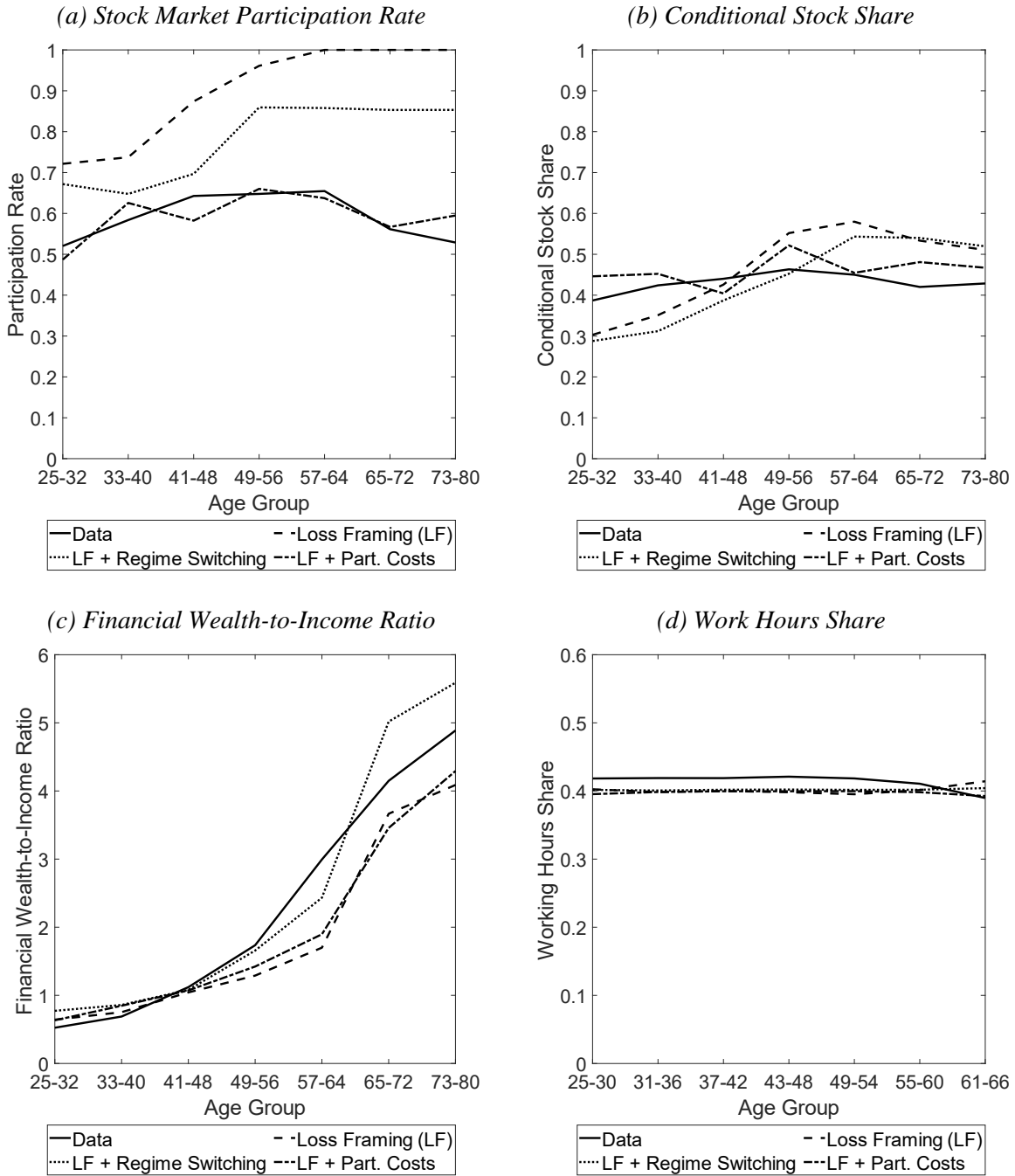*(c) Work Hours Share*

Notes: This figure compares the fit of the moments implied by the structural estimation of three life-cycle models with different preference specifications. The model moments are the ones implied by the parameter combination that gives the best fit in the structural estimation for the respective preference specifications. Those moments are compared to the empirical moments, estimated by using data from the SCF (1989–2019) and the PSID (1975–2017), and by utilizing the methodology of Deaton and Paxson (1994). The three preference specifications are loss framing (baseline model), Epstein-Zin preferences, and CRRA preferences. In each structural estimation the variables conditional stock share, financial wealth-to-income ratio, and work hours share are simultaneously targeted. The model moments are the mean values from 100,000 simulated life cycles based on optimal feedback controls.

50

Figure 3: Effect of the Loss-Framing Parameter on Portfolio Choice Decisions

(a) Stock Market Participation Rate   *(b) Conditional Stock Share*
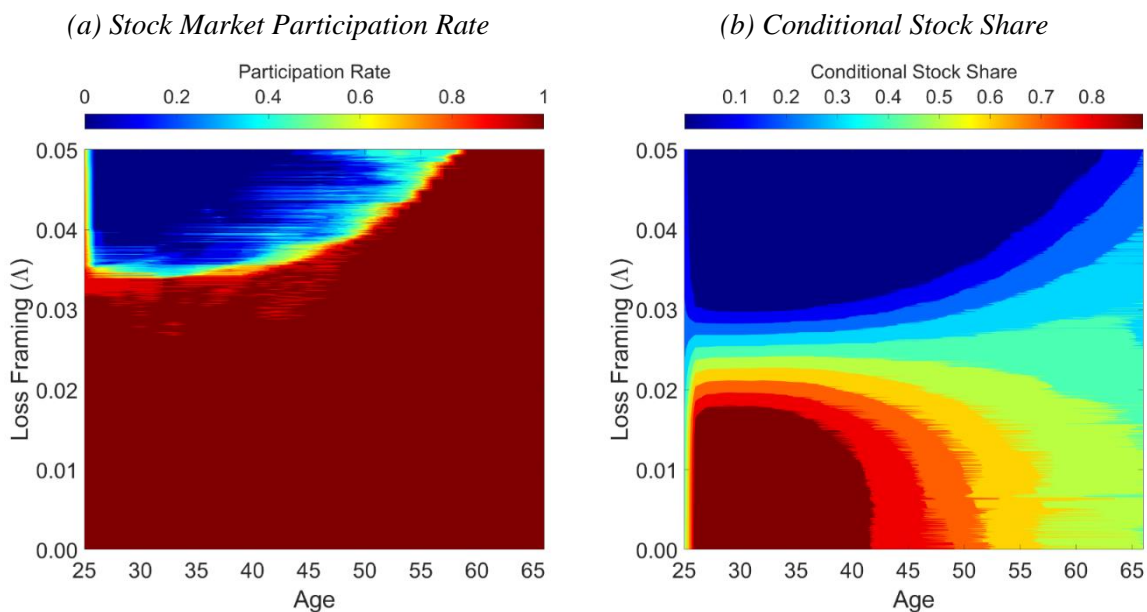
Notes: This figure illustrates the stock market participation rate and conditional stock share by age for differing values of the loss-framing parameter Λ. For an equally spaced linear grid of 1,000 parameter values for Λ ranging from 0 to 0.05, the life-cycle model for the stockholder is solved. All other preference parameter values (α, β, γ, ψ, and b) are set equal to those implied by the life-cycle model that gives the best fit in the structural estimation in which the conditional stock share, financial wealth-to-income ratio, and work hours share are simultaneously targeted (see Table 5, column 1).

Figure 4: Matching Results for the Population Models

(a) Stock Market Participation Rate

(b) Conditional Stock Share

(c) Financial Wealth-to-Income Ratio

(d) Work Hours Share

Notes: This figure compares the moments implied by the structural estimation of three life-cycle models with different model specifications for the entire population. Model moments are those implied by the parameter combination that gives the best fit in the structural estimation. Those moments are compared to the empirical moments, estimated from data from the SCF (1989–2019) and the PSID (1975–2017), and utilizing the methodology of Deaton and Paxson (1994). The models are the baseline preference specification (Loss Framing), the loss-framing model where the stock process follows a regime-switching process (Loss Framing + Regime Switching), and the loss-framing model with participation costs (Loss Framing + Participation Costs). In each structural estimation, the stock market participation rate, conditional stock share, financial wealth-to-income ratio for the population, and the work hours share are simultaneously targeted. The model moments are the mean values from 100,000 simulated life cycles based on optimal feedback controls.
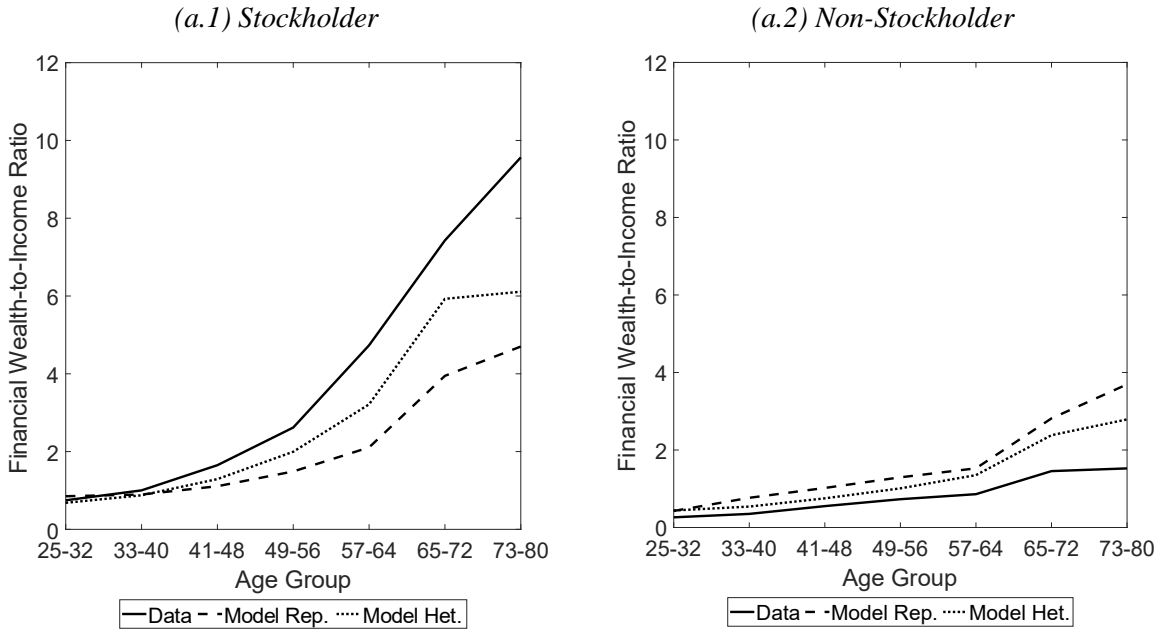
Figure 5: Matching Results for the Heterogeneous Agent Model

(a) Stock Market Participation Rate

(b) Conditional Stock Share

(c) Financial Wealth-to-Income Ratio

(d) Work Hours Share
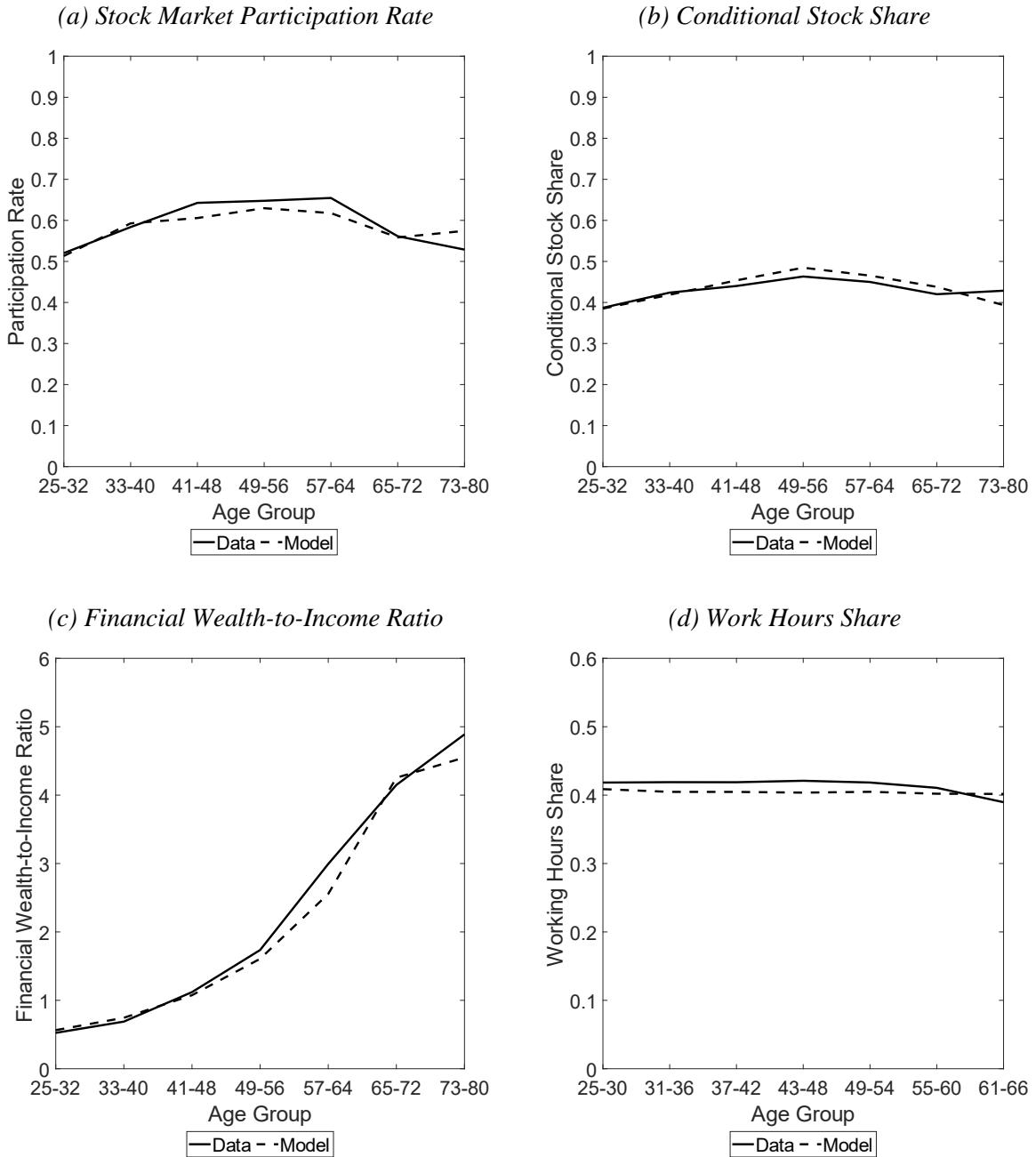
Notes: This figure shows the for each of the target variable combinations the age-dependent empirical moments using SCF and PSID data (black line) vis-a-vis the moments implied by the structural estimation of the heterogeneous agent model (dashed line), which are the mean values from 100,000 simulated life cycles scenarios based on optimal feedback controls. The three-group model consists of 30% stockholders, 20% non-stockholders, and 50% blended households, where for each group the parameters are structurally estimated for the respective group target variable combination. Subsequently all three groups are aggregated into one model. Those moments are compared to the empirical moments estimated by using data from the SCF (1989–2019) and the PSID (1975–2017), and by utilizing the methodology of Deaton and Paxson (1994).

Figure 6: Comparisons for Other Empirical Phenomena

(a) Wealth Accumulation by Investor Type

(a.1) Stockholder   *(a.2) Non-Stockholder*

(b) Portfolio Choice by Financial Wealth

(b.1) Stock Market Participation Rate   *(b.2) Conditional Stock Share*
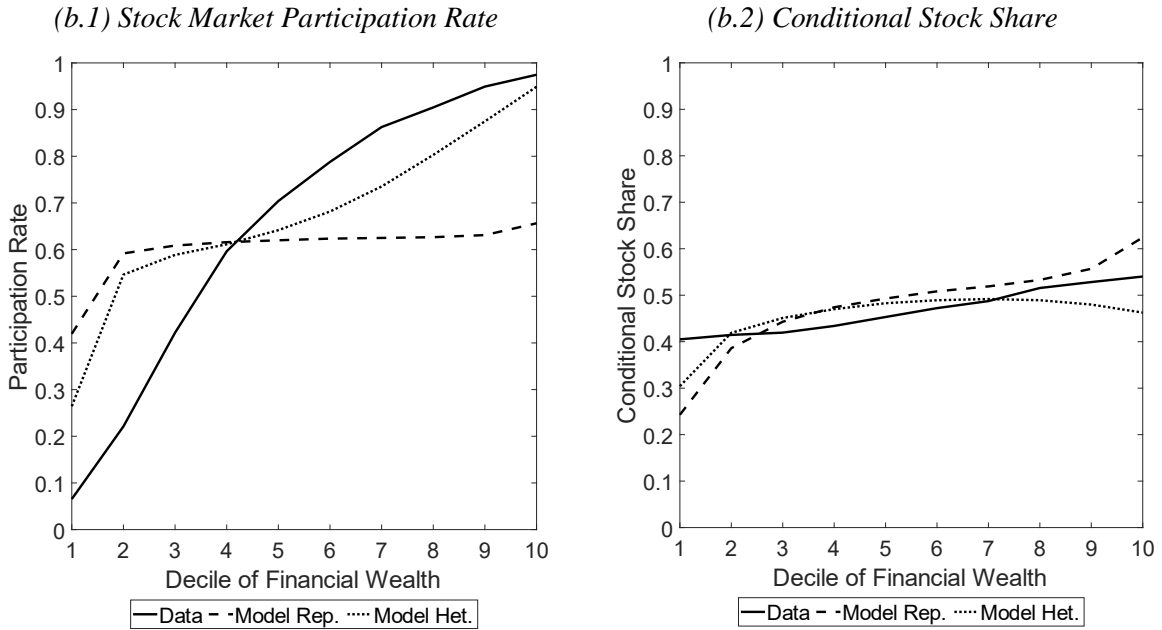
Notes: This figure compares the representative agent model with stock market participation costs (Model Rep.) with the heterogeneous agent model (Model Het.) for the financial wealth-to-income ratio for stock- and non-stockholders and the stock market participation and the conditional stock share by deciles of financial wealth measured by the dollar allocation to both stocks and bonds. The three-group model consists of 30% stockholders, 20% non-stockholders, and 50% blended households. The data are from the SCF waves from 1989 to 2019. Only households age 25 to 80 are considered. Wealth deciles for the data are calculated for each wave. The model moments are the mean values from 100,000 simulated life cycles based on optimal feedback controls.

Online Appendix

Appendix A: Relation Loss Framing and Narrow Framing with Loss Aversion

We want to show that the loss framing part with one preference parameter Λ used in our preference specification (1) is equivalent to the two-parameter (b_0, λ) approach of Barberis and Huang (2009). The following equation represents the two approaches:

$$p_t \Lambda \mathbb{E}_t[\min(G_{t+1}, 0)] = p_t b_0 \mathbb{E}_t[\max(G_{t+1}, 0) + \lambda \min(G_{t+1}, 0)]. \qquad (A.1)$$

On the left hand side, the parameter Λ controls the disutility from expected shortfalls on stock investments below a target return. On the right hand side, $b_0 > 0$ controls the strength of the narrow framing part and $\lambda > 0$ is the loss aversion parameter used in the piecewise-linear utility specification in Barberis and Huang 2009, page 1559).[27] Furthermore, p_t denotes the one-year survival probability, and G_{t+1} are stock market gains or losses relative to the benchmark return R_b:

$$G_{t+1} = S_t(R_{t+1} - R_b). \qquad (A.2)$$

Substituting (A.2) in (A.1) and dividing both sides of equation (A.1) by the non-random variables S_t and p_t, and isolating the loss-framing parameter Λ, gives the final general expression of the loss-framing parameter used in (1) in terms of the parameters (λ, b_0) used in Barberis and Huang (2009):

$$\Lambda = b_0 \left(\frac{\mathbb{E}_t[\max(R_{t+1} - R_b, 0)]}{\mathbb{E}_t[\min(R_{t+1} - R_b, 0)]} + \lambda \right) \qquad (A.3)$$

Thus, the loss-framing parameter Λ is equal to the narrow framing parameter b_0 times a ratio of two partial expectations plus the loss aversion parameter λ. This expression can be rewritten as follows ($R := R_{t+1}$):

$$\Lambda = b_0 \left(-\frac{\int_{R_b}^{\infty}(R - R_b)f(R)dR}{\int_{-\infty}^{R_b}(R_b - R)f(R)dR} + \lambda \right) = b_0 \left(-\frac{UPM_1(R, R_b)}{LPM_1(R, R_b)} + \lambda \right). \qquad (A.4)$$

The second expression illustrates that this ratio can be traced back to the common downside risk measure $LPM_1(R_b, R) = E[\max(R_b - R, 0)] = \mathbb{E}[R_b - R | R \geq R_b]\mathbb{P}(R \geq R_b)$ and the upside return measure $UPM_1(R_b, R) = E[\max(R - R_b, 0)] = \mathbb{E}[R - R_b | R < R_b]\mathbb{P}(R < R_b)$ already known in the literature (Bawa and Lindenberg 1977; Fishburn 1977; Holthausen 1981). To ensure that $\Lambda > 0$ and thus there is actual loss-framing requires that $\lambda LPM_1(R_b, R) > UPM_1(R_b, R)$. In other words, expected losses below the benchmark return multiplied by the degree of loss aversion must exceed expected gains above the benchmark.

Assuming a log-normal distribution of the gross stock return $R \sim LN(\mu, \sigma^2)$ with parameters μ and σ^2 and using Winkler et al. 1972 to calculate the partial moments expression, (A.3) has the following closed-form solution:

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[27] As pointed out by Barberis and Huang (2009), the piecewise-linear specification of the narrow framing part is closely related to Kahneman and Tversky's (1979) prospect theory and ensures that the utility function is concave and homothetic.

$$\Lambda = b_0 \left(\frac{\left(e^{\mu + \frac{\sigma^2}{2}} \frac{\Phi\left(\frac{\mu + \sigma^2 - \log(R_b)}{\sigma}\right)}{1 - \Phi\left(\frac{\log(R_b) - \mu}{\sigma}\right)} - R_b \right) \left(1 - \Phi\left(\frac{\log(R_b) - \mu}{\sigma}\right) \right)}{\left(e^{\mu + \frac{\sigma^2}{2}} \frac{\Phi\left(\frac{\log(R_b) - \mu - \sigma^2}{\sigma}\right)}{\Phi\left(\frac{\log(R_b) - \mu}{\sigma}\right)} - R_b \right) \Phi\left(\frac{\log(R_b) - \mu}{\sigma}\right)} + \lambda \right). \tag{A.5}$$

Here Φ denotes the cumulative distribution function of the standard normal distribution. Thus one easily derive a loss-framing parameter value from the other parameterizations. Table A.1 gives four calibration examples for the loss-framing parameter which further illustrates the ceteris paribus effects of the relevant input parameters for deriving the loss-framing parameter.

Table A.1: Example Calibrations for the Loss-Framing Parameter

This table presents calibration examples for the loss-framing parameter Λ. The input parameters are the narrow framing parameter b_0, the loss aversion parameter λ, the benchmark return R_b, and the stock market process parameter values μ and σ for log-normally distributed stock returns.

	Input Parameters				Output
b_0	λ	R_b	μ	σ	Λ
0.05	3.5	1.0077	0.0637	0.1564	0.0248
0.10	3.5	1.0077	0.0637	0.1564	0.0496
0.05	4.0	1.0077	0.0637	0.1564	0.0498
0.05	3.5	1.0000	0.0637	0.1564	0.0048

Appendix B: Estimation Methods and Results First-Stage Parameters

Appendix B.1: Capital Market

Since the stock index which serves as proxy for the risky asset is easily investable for retail investors, the S&P 500 total return index is chosen (data obtained from Datastream). The sample period is February 1970 to July 2020, since from then onwards, a S&P 500 total return index is available, yielding a sample of 606 monthly observations. As a real risk-free rate we use the 3-Month T-bill rate on the secondary market (data obtained from the FRED database). The sample period for the time series of the risk-free rate also spans February 1970 to July 2020. As an inflation proxy we use the monthly CPI for all urban customers with base month July 2020. We estimate the parameters of the stock process by regressing the real monthly log-returns on a constant:

$$\log(R_\tau) = \mu^\tau + \eta_\tau, \tag{B.1}$$

where the subscript τ denotes the month and the superscript τ indicates that the parameter is defined in monthly terms. Estimation results for all return parameters are depicted in B.1. The resulting annualized expected log-return amounts to $\mu = 0.0637$ with an annualized volatility $\sigma_\eta = 0.1564$, which together with the annualized gross risk-free rate $R_f = 1.0077$

results in an expected gross stock return $\mathbb{E}[R_t^{gross}] = 1.0789$ and a standard deviation $\text{Std}(R_t^{gross}) = 0.1688.$[28]

Table B.1: Estimation Results Asset Return Parameters

This table presents our estimation results for the capital market parameters. The real gross risk-free rate R_f is the mean of the 3-Month T-bill rate on the secondary market. The stock return parameters (μ, σ_η) are obtained by regressing the logarithm of the monthly real return of the S&P 500 total return index on a constant, estimated using standard maximum likelihood techniques. The sample period for each return parameter is February 1970 to July 2020, while the monthly CPI for all urban customers with base month July 2020 serves as the inflation proxy. The superscript τ on each parameter indicates that it is a monthly estimate. One, two, and three asterisks indicate that the coefficient is statistically significant at the 10%, 5%, and 1% significance level, respectively. Standard errors for each regression coefficient are displayed in parentheses.

$R_f^\tau - 1$	0.0006
μ^τ	0.0053***
	(0.0018)
σ_η^τ	0.0452***
	(0.0001)
Observations	606

Appendix B.2: Labor Income

We calibrate the wage rate process using the Panel Study of Income Dynamics (PSID) 1975-2017. The wage rate of each household is defined as the hourly wage directly reported or an inferred hourly wage rate. The inferred wage rate is obtained by dividing reported annual labor income by reported average work hours per week times 52 weeks; annual labor income is either the weekly salary times the number of weeks worked during the year or the hourly salary times 2,000 hours. Wage rate values are deflated using the CPI for all urban customers with base year 2016.[29] The data are further trimmed by dropping extreme observations below \$5 per hour or above the 99th percentile of each wave, and by dropping observations of unemployed households. Further, only ages 25-66 are considered.

To determine the deterministic component of the wage rate process, we regress the logarithm of the wage rate on a constant, a cubic age polynomial, dummies for employment status, and a set of wave dummies. The fixed effects regression function is:

$$w_{i,t} = \beta_0 + \beta_1 \frac{age}{100} + \beta_2 \left(\frac{age}{100}\right)^2 + \beta_3 \left(\frac{age}{100}\right)^3 + \beta_4 ES_{i,t} + \beta_t D_t + \epsilon_t, \qquad \text{(B.2)}$$

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[28] This estimate for the mean stock return is substantially higher while the estimate of the risk-free rate is lower than typically used value in the life-cycle literature. However, both are in line with other empirical evidence (Pagel 2018; Jorda et al. 2019). It is debatable whether to adjust expected returns by unexpected capital gains and losses, which would result in substantially lower stock returns as outlined by Fama and French (2002).

[29] The base year is 2016 and not 2017, as in each PSID wave, the questions asked to the panel participants refer to the prior year.

where $w_{i,t}$ is the log-wage rate of household i at time t, age is the age of the household in years, $ES_{i,t}$ is the employment status of the household i at time t, and D_t are wave dummies to control for year-specific shocks. The household's employment status depends on its weekly work hours and consists of one of three categories: part-time worker (below 30 hours), full-time worker (between 30 and 40 hours) and over-time worker (above 40 hours). Estimation results are depicted in Panel A of Table B.2. Resulting wage rates illustrate the typical hump-shaped form of the wage rate over the life cycle.

We estimate the variances of the permanent and transitory components of the wage process using the decomposition procedure in Carroll and Samwick (1997) and Cocco et al. (2005), by regressing the variance of each household's stochastic component of the log wage on the yearly difference between observations and a constant. Here we make use of the maximum number of yearly log wage differences resulting in differences from one to 42 years. Results are depicted in Panel B of Table B.2. The variance for the permanent wage shock is 0.0080 and 0.0606 for the transitory wage shock, both in line with other estimates in the literature.

To estimate the contemporaneous correlation of the logarithm of the aggregate component of stochastic permanent wage $\log(\xi_t)$ with the innovation of the logarithm of the stock return η_t, we follow the procedure in Campbell et al. (2001) and Cocco et al. (2005). Thus we extract the correlation coefficient from the regression coefficient of regressing the cross-sectional mean of the household one-year log wage differences of each wave against the demeaned stock return of the same year without a constant. We also follow Angerer and Lam (2009) by assuming that the first differences in log wages in two adjacent years are equal when the survey is done biennially. Results appear in Panel C of Table B.2; here the regression coefficient is strikingly low and statistically insignificant, resulting in insignificant correlation coefficients. Consequently, the hypothesis of zero correlation cannot be rejected, which is why we assume $\rho_{\log(\xi),\eta} = 0$.[30]

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[30] This result also holds for lags in the relation between the two innovations.

Table B.2: Estimation Results for the Wage Rate Process

This table presents estimation results for the wage rate process parameters. Panel A displays the results of fixed-effects regressions of the logarithm of the real hourly wage rate on a set of age dummies, dummies for working part-time (under 20 hours per week) or over-time (over 40 hours a week), and wave dummies (not reported here). The data are taken from the PSID (1975-2017). The wage per hour is defined the sum of household labor income divided by the average work hours that year. The sample is restricted to employed respondents aged 25-66, and observations are excluded if hourly wages less than \$5 or greater than the 99th percentile of each wave. Panel B displays the results of the regression for the variance decomposition of the stochastic component of the wage process. The estimation procedure for the error structure follows Carroll and Samwick (1997) by regressing the variance of each household's stochastic component of log-wage on the yearly difference between observations and a constant. Estimation results make use of the maximum number of available wage differences of up to 42 years. Panel C shows the results of the estimation of the correlation coefficient between the logarithm of the permanent aggregate wage shock and the stock return shock. The estimation procedure follows Campbell et al. (2001) by extracting the correlation coefficient from regressing the cross-sectional mean of the household one-year log wage differences of each wave against the demeaned stock return of the same year without a constant. All pecuniary values are deflated using the CPI for all urban customers with base year 2016. One, two, and three asterisks indicate that the coefficient is statistically significant at the 10%, 5%, and 1% significance level, respectively. Robust standard errors for each regression coefficient are displayed in parentheses.

Panel A: Deterministic Profile	
Constant	1.6428***
	(0.0318)
Age/100	6.4680***
	(0.2380)
$(Age/100)^2$	-9.2595***
	(0.5813)
$(Age/100)^3$	3.3715***
	(0.4574)
Discount Part-Time	-0.1979***
	(0.0024)
Premium Over-Time	0.2126***
	(0.0010)
Observations	1,317,891

Panel B: Shocks	
Permanent (σ_ϑ^2)	0.0080***
	(0.0000)
Transitory (σ_φ^2)	0.0606***
	(0.0002)
Observations	1,702,532

Panel C: Correlation Labor Income and Stock Return Shocks	
Regression Coefficient	0.0130
	(0.0156)
$\rho_{\log(\xi),\eta}$	0.1291
Observations	42

Appendix B.3: Retirement Income

We calibrate the parameters of retirement income using PSID data. We assume that gross retirement income is the household's reported total Social Security benefits received for those years when it stated it has been retired, using waves from 1986-2017. We restrict the sample to those age 60+. All values are deflated using the CPI for all urban customers with base year 2016.

The benefit replacement ratio for each household is defined as the ratio of the reported Social Security benefits over all available time periods to the assumed deterministic average labor income from age 25-66 given by our wage rate estimation assuming $\bar{N} = 0.4$, and $\Upsilon = 5200$, which gives on average 40 weekly work hours over the household's lifetime. The estimate of the economy-wide ratio is the mean across all individual ratios. The resulting benefit replacement ratio is displayed in Panel A of Table B.3 and amounts to 0.5838.

Table B.3: Estimation Results: Retirement Income Process

This table presents the estimation results of the retirement income process parameters. The data are taken from the 1986 to 2017 waves of the Panel Study of Income Dynamics (PSID). Panel A displays the results for the benefit replacement rate, defined as the average across all households of the fraction of each household's average reported social security benefits divided by the assumed deterministic average labor income from age 20-66 for an average 40 weekly work hours over the household's lifetime. Panel B shows results for the estimates of the average proportion of out-of-medical expenditures, defined as the fraction of the sum of doctor expenditures, nursing home and hospital bills, and prescription expenditure, divided by the same year's gross retirement income, from the PSID (waves 1999 to 2017). Panel C shows the results of the regression for the variance decomposition of the stochastic component of gross retirement income process net of medical expenditures. The estimation procedure for the error structure follows Carroll and Samwick (1997). All pecuniary values are deflated using the CPI for all urban customers with base year 2016. One, two, and three asterisks indicate that the coefficient is statistically significant at the 10%, 5%, and 1% significance level, respectively. Robust standard errors for each regression coefficient are displayed in parentheses.

Panel A: Benefit Replacement Ratio	
ζ	0.5838
Observations	11,650
Panel B: Proportion of Out-of-Pocket Medical Expenditures	
λ	0.1175
Observations	18,435
Panel C: Shocks	
Transitory (σ_ε)	0.0903***
	(0.0023)
Observations	11,430

The average portion of retirement income the household must spend on medical expenses λ is estimated by dividing the reported out-of-pocket medical expenditures from the PSID, composed of the sum of doctor expenditures, nursing home and hospital bills, and prescription expenditures divided by that year's gross retirement income. The mean across all observations

is then our estimate. The data come from the PSID (1999-2017), since for these waves the reported medical costs have a consistent definition. We trim the data for high ratios above the 99th percentile. The resulting value is displayed in Panel B of Table B.3 and equals 0.1175.

We estimate the variances of the stochastic component of retirement income net of medical expenditures using the same decomposition procedure as employed in the wage rate process, only that instead of the wage rate, gross retirement income adjusted for out-of-pocket medical expenditures is used. We restrict the sample to households age 60-90. The result is depicted in Panel C of Table B.3. We set the permanent shock to zero and the transitory shock amounts to 0.0903.

Appendix B.4: Housing Costs

Following Gomes and Michaelides (2005) and Love (2010), we define the individual housing costs for each household i in year t as the ratio of annual mortgage payments and rental costs H to gross labor or retirement income:[31]

$$h_{i,t} \equiv \frac{H_{i,t}}{Y_{i,t}^{gross}}. \tag{B.3}$$

The data are PSID (1999-2017) given the consistent definition of housing expenditures across those waves. Annual gross labor income and gross retirement income are defined in the same way as before. We restrict the sample to households age 25-85. We further eliminate observations with a ratio below zero or above one. To obtain housing costs, we regress the individual housing expenditure ratio on a cubic age polynomial and a set of wave dummies. The fixed-effects regression is defined as follows:

$$h_{i,t} = \beta_0 + \beta_1 \frac{age_{i,t}}{100} + \beta_2 \left(\frac{age_{i,t}}{100}\right)^2 + \beta_3 \left(\frac{age_{i,t}}{100}\right)^3 + \beta_t D_t + \epsilon_t. \tag{B.4}$$

We further winsorize the resulting housing costs function $h(t)$ at zero for negative values of the fitted polynomial, i.e.:

$$h(t) = \max \left(\beta_0 + \beta_1 \frac{age_t}{100} + \beta_2 \left(\frac{age_t}{100}\right)^2 + \beta_3 \left(\frac{age_t}{100}\right)^3, 0 \right). \tag{B.5}$$

Estimation results are depicted in Table B.4. The resulting function is decreasing in age and winsorization of the housing costs is required for households age 95 and above.

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[31] In fact, only a few households pay mortgages and rent simultaneously.

Table B.4: Estimation Results for Housing Costs

This table presents estimation results for housing costs. The data are from the Panel Study of Income Dynamics (PSID) from 1999 until 2017. For each household and in each wave, we compute the ratio of annual mortgage payments plus rent payments relative to annual gross labor income or gross retirement income, and regress this ratio against a constant, a cubic polynomial of age, and wave dummies (not reported). We eliminate all observations with age below 25 and over 85, as well as ratios below zero and above one. One, two, and three asterisks indicate that the coefficient is statistically significant at the 10%, 5%, and 1% significance level, respectively. Robust standard errors for each regression coefficient are displayed in parentheses.

Constant	0.5212^{***}
	(0.0148)
Age/100	-1.5218^{***}
	(0.0971)
$(\text{Age}/100)^2$	2.6889^{***}
	(0.2014)
$(\text{Age}/100)^3$	-1.7591^{***}
	(0.1320)
Observations	233,948

Appendix B.5: Starting Financial Wealth-to-Income Ratios

Initial financial wealth is defined as a multiple of the household's first-year gross income; the distribution fits the observed financial wealth-to-income ratio of households age 18 to 25 in the Survey of Consumer Finances (SCF) for stockholders, non-stockholders, and the overall population. The respective financial wealth-to-income ratios are defined as the sum of the reported stock and bond allocations divided by gross labor income. This distribution is fitted to a generalized Pareto distribution using maximum likelihood estimation. We restrict the density to positive values only and further winsorize the right tail of each distribution to mitigate the effect of extremely high ratios on the mean. Moreover, we truncate the fitted distribution in order to ensure that the resulting mean financial wealth-to-income ratio is below the ratio of the first age group in the structural estimation. Resulting parameter values are reported in Table B.5, where μ is the location parameter, σ is the scale parameter, and ξ is the shape parameter of the generalized Pareto distribution.

Table B.5: Estimation Results: Starting Financial Wealth-to-Income Ratios

This table presents estimation results for the starting financial wealth-to-income ratios (in simulations) for stockholders, non-stockholders, and the general population. Data are from the SCF on households age 18-25 having financial assets and positive labor income. The financial wealth-to-income ratio is defined as the sum of the reported stock and bond allocation divided by gross labor income. The resulting ratios are pooled and fitted to a generalized Pareto distribution using maximum likelihood estimation. μ denotes the location parameter, σ denotes the scale parameter, and ξ denotes the shape parameter of the generalized Pareto distribution.

	Stockholders	Non-Stockholders	Population
μ	0.0000	0.0000	0.0000
σ	0.3707	0.0751	0.1168
ξ	0.7115	1.0754	1.0618
Observations	2,960	7,742	10,702

Appendix B.6: Regime-Switching Stock Process

To estimate stock return parameters needed for the model, where the stock returns follows a two-dimensional Markovian regime-switching process, we use the yearly S&P 500 total return index from 1929 to 2020 as a proxy for the risky asset. The returns are deflated using the annual CPI for all urban customers with base year 2020. The parameters of the regime-switching model are driven by the stochastic state variable $s_\tau \in \{1,2\}$, are estimated using the following model:

$$\log(R_t) = \mu_{s_t} + \epsilon_t, \text{ with } \epsilon_t \sim N\left(0, \sigma_{s_t}^2\right) \text{ and } s_t \in \{1, 2\}, \tag{B.6}$$

where the subscript t denotes the year of the observation in the sample period. The parameters are estimated using the expectation-maximization (EM) algorithm by Dempster et al. (1977) and Hamilton (1990). The standard errors are obtained by bootstrapping the data using 1,000 resamples. The estimation results for all return parameters are depicted in Table B.6. The expected annual log-returns amount to -0.1754 and 0.1412 for the first and second regime, respectively, with corresponding volatilities of 0.2116 and 0.1313. Thus, the first regime is a bear market with negative returns and heightened volatility levels, while the second regime is a bull market with positive returns and modest volatility. The resulting transition probability matrix is as follows:

$$\mathbf{P} = \begin{bmatrix} 0.4532 & 0.5468 \\ 0.0940 & 0.9060 \end{bmatrix}, \tag{B.7}$$

while the invariant distribution vector is equal to:

$$\boldsymbol{\pi} = \begin{bmatrix} 0.1467 \\ 0.8533 \end{bmatrix}. \tag{B.8}$$

Consequently, the Markov chain governing the regime-switching behavior of the stock process is further ergodic.

During the simulation of the life-cycle model, the starting financial regime is determined by the invariant distribution vector $\boldsymbol{\pi}$ and is randomly assigned to the households, whereas the regime sequence for each household is governed by the two-dimensional Markov chain implied by the transition probability matrix $\mathbf{P}$.

Table B.6: Estimation Results: Regime-Switching Stock Process

This table presents estimation results for the regime-switching stock process parameters. The times series used is the logarithm of the annual real return on the S&P 500 total return index for the sample period 1929-2020, where the annual CPI for all urban customers with base year 2020 serves as inflation proxy. The subscript under each parameter denotes the number of the respective regime. The parameters are estimated using the EM algorithm. One, two, and three asterisks indicate that the coefficient is statistically significant at the 10%, 5%, and 1% significance level, respectively. Standard errors for each regression coefficient are displayed in parentheses and are obtained by bootstrapping the sample data using 1,000 resamples.

μ_1	-0.1754*
	(0.0906)
μ_2	0.1412***
	(0.0396)
σ_1	0.2116***
	(0.0286)
σ_2	0.1313***
	(0.0339)
Observations	92

Appendix B.7: Scaling Factor for Stock Market Participation Costs

The age-dependent scaling factor F_t is determined using the American Time Use Survey (ATUS) microdata from the Bureau of Labor Statistics (2003-2019). Participation costs are proxied by the household's reported average time spent for "Financial Management" (variable code: 020901), where activities related to the acquisition of stock market information are a subset of the reported activities in that category. We only consider households age 25-80 and take the mean of the reported time for each age. The resulting age-dependent times are fitted using a first-order polynomial function that gives the best fit for the data in a least-squares sense. For each age, the resulting function is divided by the function value for a household age 66, which is the last age before retirement at time period K. This results in the following scaling factor:

$$F_t = 0.0152(t + 24). \tag{B.9}$$

Appendix C: Empirical Targets

We use two datasets to generate the empirical targets for the structural estimation. All portfolio and wealth-related target variables are obtained from Survey of Consumer Finances (SCF), a triennial survey on the financial assets of U.S. households. For the work hours we use the Panel Study of Income Dynamics (PSID), which we also use to estimate the labor and retirement income-related first-stage parameters.

For the SCF, we use the waves from 1989 to 2019 given the consistent variable definitions across those waves. We scale the individual weights within each wave such that the cumulative

weights are the same across waves and further deflate all variables to 2019-dollar values. We delete observations with no financial assets and observations with zero or negative labor income. For the PSID, we use the same sample for which we estimate the wage process (1975-2017). The observations included are those of employed individuals with an hourly wage rate above $5 per hour or below the 99th percentile of each wave.

Furthermore, we group each observation of each variable in age buckets. This is motivated by greater data availability compared to single-age groupings; it further has the advantage of some data smoothing. To give each targeted variable the same weight, we use an equal number of age groups, namely seven for each variable. For all variables, the starting age is 25, while the ending age is 80 for the SCF-based variables and 66 for the PSID-based variable.

For the SCF, we rely on the following variable definitions. The allocation in stocks and financial wealth are defined as in the public summary of the SCF data. Consequently, the allocation to stocks includes direct stockholdings, investments in stock mutual funds, and the stock allocation of IRAs or Keogh plans, thrift-type retirement accounts, savings account, and other managed assets with equity interest. Financial wealth is the sum of transaction accounts, certificates of deposit, pooled investment funds, savings bonds, directly held stocks, directly held bonds, cash value of whole life insurances, annuities, trusts, quasi-liquid retirement accounts, and other miscellaneous financial assets. Following Gomes and Michaelides (2005) and Peijnenburg (2018), annual labor or retirement income is defined as the sum of the income from wages, salaries, Social Security, and other pensions in order to mimic the labor income definition of the PSID as closely as possible. Work hours are reported weekly work hours in the PSID.

The definitions of the targeted empirical moments are then as follows. Stock market participation is equal to one if the total dollar stock allocation is above zero, zero else. The stock share conditional on stock market participation is the fraction of total stock allocation to total financial wealth given a non-zero allocation to stocks. The financial wealth-to-income ratio is defined as the ratio of total financial wealth to annual labor or retirement income. The financial wealth-to-income ratio for stockholders is defined as the ratio given a non-zero allocation to stocks, while the financial wealth-to-income ratio for non-stockholders is defined as the ratio given a zero allocation to stocks. We also trim the data by deleting very high observations with financial wealth-to-income ratios above 100 for a more robust estimate. The work hours share is defined as the average weekly work hours reported in PSID divided by our assumed 100 weekly waking hours.

Appendix D: SMM Estimation Methodology

Appendix D.1: Goodness-of-Fit

The moments of the simulated data are computed analogously to those from the data. Consequently, household i is defined as participating in the stock market at time period t if its stock allocation is above zero:

$$y_{i,t}^{stock\ participation} = \mathbb{1}_{\{S_{i,t}>0\}}. \tag{D.1}$$

The stock share conditional on participation is defined as the ratio of the dollar allocation to stocks over total financial wealth, conditional on a non-zero allocation to stocks:

$$y_{i,t}^{cond.\ stock\ share} = \frac{S_{i,t}}{W_{i,t}}\bigg|_{S_{i,t}>0}. \tag{D.2}$$

The financial wealth-to-income ratio for a stockholder is defined as the ratio of financial wealth to gross labor or retirement income, conditional on a non-zero allocation to stocks:

$$y_{i,t}^{fin-inc\ ratio\ stockholder} = \frac{W_{i,t}}{Y_{i,t}^{gross}}\bigg|_{S_{i,t}>0}. \tag{D.3}$$

The financial wealth-to-income ratio for non-stockholders is defined as the ratio of financial wealth to gross labor or retirement income, conditional on a zero allocation to stocks:

$$y_{i,t}^{fin-inc\ ratio\ non-stockholder} = \frac{W_{i,t}}{Y_{i,t}^{gross}}\bigg|_{S_{i,t}=0}. \tag{D.4}$$

The financial wealth-to-income ratio for the population is defined as the ratio of financial wealth to gross labor or retirement income, unconditional on the asset allocation:

$$y_{i,t}^{fin-inc\ ratio\ population} = \frac{W_{i,t}}{Y_{i,t}^{gross}}. \tag{D.5}$$

The work hours share is defined as the share of waking time devoted to work:

$$y_{i,t}^{work\ share} = N_{i,t}. \tag{D.6}$$

The model moment for each target j of age group a is defined as the mean of the variable y for each time period t for age group a, which has an age width of $\Delta\tau_j$ and a mean across the total simulated population:

$$m\big(y_a^{j,model}\big) = \frac{1}{N\Delta\tau_j}\sum_{i=1}^{N}\sum_{t=\tau_a}^{\tau_a+\Delta\tau_j} y_{i,t}^{j,model}, \tag{D.7}$$

where N is the number of simulated life-cycle scenarios of the household and τ_a is the lowest age in age group a. We define the vector of moments $\boldsymbol{m}$ for each target variable j as follows:

$$\boldsymbol{m}\big(y^{j,model}\big) = \begin{bmatrix} m\big(y_1^{j,model}\big) \\ \vdots \\ m\big(y_n^{j,model}\big) \end{bmatrix}, \tag{D.8}$$

where n denotes the number of age groups. We further concatenate the moment vector across the different target variables to form a final moment vector $\boldsymbol{m}(\boldsymbol{y}^{model})$, whose length depends on the exact combination of targeted variables.

The parameter combination vector of the continuous model parameters is defined as follows:

$$\boldsymbol{\theta} \equiv [\alpha\ \beta\ \gamma\ \psi\ b\ \Lambda\ \chi]. \tag{D.9}$$

The length of the parameter vector depends on the model specification i, i.e.: [32]

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[32] For the rest of the section, we refrain from indicating that the parameter vector is model-dependent.

$$\boldsymbol{\theta}^i \subseteq \boldsymbol{\theta}. \tag{D.10}$$

The SMM estimator is then the preference parameter combination that minimizes the distance function, which is defined as the squared percentage deviation of the difference in the vector of simulated model moments from the targeted data moments:

$$\boldsymbol{\theta}_{SMM} = \arg\min_{\boldsymbol{\theta}} \left[\frac{\boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}) - \boldsymbol{m}(\boldsymbol{y}^{data})}{\boldsymbol{m}(\boldsymbol{y}^{data})} \right]^{\top} \boldsymbol{W} \left[\frac{\boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}) - \boldsymbol{m}(\boldsymbol{y}^{data})}{\boldsymbol{m}(\boldsymbol{y}^{data})} \right]. \tag{D.11}$$

We use the percentage deviation of the moments given the different scale of the target variables. Furthermore, the weighting matrix $\boldsymbol{W}$ is equal to the identity matrix $\boldsymbol{I}$ as we combine moments from two data sets, namely the SCF and the PSID and thus we cannot use the typical modeling approaches, such as using inverse of the variance-covariance matrix of the actual data moments (Gourieroux et al. 1993) or bootstrapping the actual data (Hall and Horowitz 1996). This results in a consistent but inefficient estimate of the parameters.

Appendix D.2: Standard Errors

To overcome the efficiency problem that the use of the identity matrix as weighting matrix poses, we calculate the standard errors as in Fagereng et al. (2017) by bootstrapping the simulated data. We first simulate 500,000 paths of the life-cycle model for the given optimal parameter combination $\boldsymbol{\theta}_{SMM}$ and subsequently draw $B = 1,000$ independent samples with replacement of size $N = 100,000$ households, which corresponds to the sample size of the simulation used to obtain the SMM estimate. We then compute the moments from each sample and use the B realizations of these moments to calculate an estimate of their variance-covariance matrix $\boldsymbol{W}_{SMM}$:

$$\boldsymbol{W}_{SMM}(\boldsymbol{\theta}_{SMM}) = \frac{1}{B} \sum_{b=1}^{B} \left[\boldsymbol{m}_b(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM}) - \frac{1}{B} \sum_{b=1}^{B} \boldsymbol{m}_b(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM}) \right]$$
$$\left[\boldsymbol{m}_b(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM}) - \frac{1}{B} \sum_{b=1}^{B} \boldsymbol{m}_b(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM}) \right]^{\top}, \tag{D.12}$$

where $\boldsymbol{m}_b(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM})$ is the moment vector implied by the b-th resample of the simulation for the life-cycle model with preference parameter vector $\boldsymbol{\theta}_{SMM}$. The asymptotic covariance matrix is then given by the following expression:

$$\boldsymbol{Q}(\boldsymbol{W}_{SMM}) = \left(1 + \frac{1}{B}\right) \left[\frac{\partial \boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM})^{\top}}{\partial \boldsymbol{\theta}_{SMM}} (\boldsymbol{W}_{SMM})^{-1} \frac{\partial \boldsymbol{m}(\boldsymbol{y}^{model}|\boldsymbol{\theta}_{SMM})}{\partial \boldsymbol{\theta}_{SMM}} \right]^{-1}, \tag{D.13}$$

where the derivatives of the moments with respect to the parameters are approximated using a central differences scheme. The incremental increase and decrease of the i-th element of parameter vector $\boldsymbol{\theta}$ is $\delta_i = \max(|\boldsymbol{\theta}_i|, 1)\sqrt[3]{\varepsilon}$, where $\varepsilon = 2.2204e - 16$ is the machine epsilon of our programming language MATLAB. The standard errors are then obtained by taking the square root of the diagonal elements of matrix $\boldsymbol{Q}$, i.e.:

$$\boldsymbol{SE}_{\boldsymbol{\theta}_{SMM}} = \sqrt{\text{tr}(\boldsymbol{Q})}. \tag{D.14}$$

Appendix D.3: Determination of Estimates

The overall objective of the structural estimation is to find the parameter combination vector $\boldsymbol{\theta}$ that solves the subsequent minimization problem:

$$\min_{\boldsymbol{\theta}\in\Omega} f(\boldsymbol{\theta}) \quad \text{with } \Omega = \{\boldsymbol{\theta} \in \mathbb{R}^k : \boldsymbol{\theta}^l \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}^u\}, \tag{D.15}$$

where the objective function is equivalent to the criterion function of the SMM estimation and is defined as follows:

$$f(\boldsymbol{\theta}) = \left[\frac{m(y^{model}|\boldsymbol{\theta})-m(y^{data})}{m(y^{data})}\right]^\top I \left[\frac{m(y^{model}|\boldsymbol{\theta})-m(y^{data})}{m(y^{data})}\right], \tag{D.16}$$

and k is the dimension of parameter combination vector $\boldsymbol{\theta}$ and $\boldsymbol{\theta}^l$ and $\boldsymbol{\theta}^u$ denote the lower and upper parameter vectors, respectively.

The constraint set Ω for all parameters is constructed using a structural approach. The starting points for the upper and the lower bounds are values previously used in the life-cycle literature. These two values are linearly interpolated on an equidistant 10-point grid and further linearly extrapolated in both directions for seven further equally distant points. This approach ensures that our constraint set is consistent with respect to each parameter given and, to a certain extent, literature-based. To determine the baseline loss-framing parameter, we convert the narrow framing and loss aversion parameter of the respective study together with our baseline stock market parametrization and our benchmark return to the loss framing value. For the model in which the stock follows a regime-switching process, we use the parameters of the stress regime 1 for the upper bound. The participation costs for the upper bound are translated by using the parameter value of Khorunzhina (2013) for CRRA preferences (equal to 0.044), which is multiplied by our average yearly labor income $\bar{Y}$ to obtain yearly costs. We impose several constraints on the parameter vector to ensure that economically unreasonable values are excluded. We restrict the subjective discount factor to be less than one, the EIS to be above zero and above the inverse of the maximum of the risk aversion parameter, the risk aversion term to be above one; we also require the bequest motive, the loss-framing strength, and the stock market participation costs to be equal to or greater than zero. The numerical value that satisfies these bounds up to the fourth decimal point serves as the ultimate bound for the respective parameter value in the grid. The literature-based parameter values, their sources, and the final extrapolated values are displayed in Table D.1.

Similar to Catherine (2022) we perform a two-step approach for determining the best-fitting parameter combination. In the first step, we carry out an exhaustive discrete parameter grid search over the entire parameter grid specified as before. The reason why we do not employ a global optimization algorithm[33] in the first stage lies in the flexibility of our approach: we need to solve the model for all parameter combinations only once, while being able to extract a variety of empirical target combinations and specifications thereof. This allows us to perform the estimation for a given model specification for all relevant target variable combinations. Another advantage of this approach is that each parameter combination run is completely

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[33] See Rios and Sahinidis (2013) for a review and comparison of derivative-free global and local optimization algorithms.

independent from the other runs, which allows the utilization of large-scale parallelization techniques over the parameter combinations to perform the entire estimation. This approach additionally allows us to split the problem and do the optimization runs in several batches, consequently reducing the risk of losing results. To obtain the parameter grid, we first construct a quasi-random point set from a Sobol sequence Ξ of length n with a dimension of k, where k is equal to the number of continuous model parameters in vector $\boldsymbol{\theta}$. We additionally skip the first 100 points of the sequence to mitigate the problem of an unbalanced sequence. This point set is multiplied by the upper and lower points of our parameter vectors resulting in the grid of parameter combinations implied by the Sobol sequence:

$$\boldsymbol{\Theta}^s = \Xi \odot \boldsymbol{\Theta}^u + \left(\boldsymbol{J}_{n,k} - \Xi\right) \odot \boldsymbol{\Theta}^l, \tag{D.17}$$

where $\odot$ is the Hadamard product, $\boldsymbol{J}_{n,k}$ is an all-ones matrix of dimension $n \times k$, and $\boldsymbol{\Theta}^u$ and $\boldsymbol{\Theta}^l$ are the parameter combination matrices that contain in each row the upper and lower parameters vector, respectively. The resulting parameter combination matrix $\boldsymbol{\Theta}^s$ is further supplemented by the loss benchmark parameter R_b, which is set to R_f, as the other economically reasonable value of one would be the same as upscaling the loss-framing parameter. The length n_i of the Sobol sequence for model specification i is

$$n_i = \min(7.5^{k_i}, 200{,}000), \tag{D.18}$$

where k_i is the dimension of the continuous parameter vector of model specification i. Furthermore, each n_i is roughly rounded to some reasonable numerical value, which results in a sequence of length 3,500, 25,000, 200,000, and 200,000, for models with four, five, six, and seven parameters, respectively.[34]

In the second step, we use the three best-fitting parameter combinations for each empirical target combination from the first step and run for each one a local optimization, where the respective parameter combination from the first step serves as the starting point. For the local optimization, we use a direct search algorithm, as it has several desirable properties for our purposes: it works for black-box optimization problems, it has a predetermined starting point, and it allows to some degree the utilization of parallelization techniques. As a search method we use a Mesh Adaptive Direct Search (MADS) algorithm (Audet and Dennis 2006; Abramson et al. 2009), which introduces randomness in the mesh construction and as poll method we use a Generalized Pattern Search (GPS) algorithm (Torczon 1997; Lewis and Torczon 1999), which uses a deterministic mesh construction. Both algorithms evaluate the objective function at a finite number of set points on a mesh in order to find an improved mesh point. We parallelize over all available directions of the mesh, which is dependent on the position of the parameter combination in the search space. We further set the mesh expansion to a value of 1.5 and the mesh contraction to a value of 2/2.5. We allow the algorithm to iterate up to 100 times, while at each iteration we use the maximum possible number of function evaluations. During the second step, the bounds of the parameter grid from the first step are still in effect. The best-

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[34] To counterbalance the shorter sequence length of the seven-parameter models, we perform the second step for those models twice.

fitting parameter combination of all runs is considered our global optimum and hence the estimate for the respective target moment combination and model.

We further impose the constraint that there is only a preference for the early resolution of uncertainty with respect to the consumption-leisure stream. This is theoretically implied by Kreps and Porteus (1979) and Machina (1984) and supported by experimental evidence of Brown and Kim (2014) and Meissner and Pfeiffer (2022). Consequently, we do not perform the estimation for each parameter combination for which $\gamma < 1/\psi$ holds.

Table D.1: Upper and Lower Bounds: Second-Stage Parameter Grid

This table displays the upper and lower bounds for each of the continuous model parameters for the parameter grid for the first step during the determination of the SMM estimates. The literature-based lower and upper bounds with the respective source are shown for each parameter. To determine the baseline loss-framing parameter, we convert the narrow framing and loss aversion parameter of the respective study together with our capital market parametrization and our benchmark return to the loss-framing value. We proceed analogously for the loss-framing parameter Λ_{RS} in the regime-switching model, except that for the upper bound, the stock return parameters for the stress regime (regime 1) are used. Participation costs are in $1,000. Participation costs for the upper bound are translated using the parameter value 0.044 of Khorunzhina (2013), and then multiplied by our average yearly labor income $\bar{Y}$. The extrapolated bounds, which serve as the final lower and upper bounds of the parameter grid, are obtained by first linearly interpolating the literature-based bounds on an equidistant 10-point grid and then extrapolating the function in both directions for seven further equally-distant points. The values displayed are the end values for both directions. We restrict the subjective discount factor to be less than one, the risk aversion to be above one; additionally the bequest motive, the loss-framing strength, and the stock market participation costs must be greater than or equal to zero. We further allow for a preference of early resolution of uncertainty, which is why the inverse of the EIS must be less than the coefficient of risk aversion.

| Parameter	Literature-Based Bounds				Extrapolated Bounds	
	Lower Bound	Source	Upper Bound	Source	Lower Bound	Upper Bound
β	0.90	Li and Smetters (2011)	0.99	Gomes et al. (2022)	0.8300	0.9999
α	0.9	Kim et al. (2016)	1.3	Chai et al. (2011)	0.5889	1.6111
ψ	0.2	Love and Phelan (2015)	1.0	Campbell et al. (2001)	0.0989	1.6222
γ	3.0	Cocco and Gomes (2012)	10.0	Cocco et al. (2005)	1.0001	15.4444
b	1.0	Gomes and Michaelides (2005)	5.0	Polkovnichenko (2007)	0.0000	8.1111
Λ	0.0010	Chai and Maurer (2012)	0.0248	Calvet et al. (2022)	0.0000	0.0444
Λ_{RS}	0.0010	Chai and Maurer (2012)	0.1678	Calvet et al. (2022)	0.0000	0.2987
χ	0.3000	Fagereng et al. (2017)	1.6972	Khorunzhina (2013)	0.0000	2.7839

Appendix E: Additional Tables and Figures

Table E.1: Model vs. Data Moments: Heterogeneous Agent Models

This table compares the moments implied by the structural estimation and the empirical moments for each group of the heterogeneous agent model. The stockholder (30%) and non-stockholder (20%) group match their standalone target variables, while the blended group (50%) is estimated such that it minimizes the residual distance needed for the combined model to match the population moments. Data moments are estimated using data from the SCF (1989–2019) and the PSID (1975–2017), and by utilizing the methodology of Deaton and Paxson (1994). The model moments are the ones implied by the parameter combination that gives the best fit in the respective structural estimation and are the mean values from 100,000 simulated life cycles. Each of the seven age groups for the stock participation rate, the conditional stock share, and the financial wealth-to-income ratio consist of eight consecutive years of life, starting from 25 and ending with 80. Each age group for the work hours share uses six consecutive years of life, starting from 25 and ending with 66.

	Age Group						
	1	2	3	4	5	6	7
Panel A: Stockholder							
Panel A.1: Conditional Stock Share							
Data	0.3870	0.4240	0.4401	0.4632	0.4500	0.4201	0.4286
Model	0.3909	0.4264	0.4406	0.4539	0.4398	0.4218	0.4378
Panel A.2: Financial Wealth-to-Income Ratio							
Data	0.7471	1.0026	1.6499	2.6184	4.7334	7.4298	9.5641
Model	0.7731	1.0298	1.5832	2.6962	4.6735	8.3233	8.0885
Panel A.3: Work Hours Share							
Data	0.4184	0.4190	0.4189	0.4210	0.4184	0.4107	0.3897
Model	0.4338	0.4076	0.4084	0.4117	0.4115	0.4102	0.4202
Panel B: Non-Stockholder							
Panel B.1: Financial Wealth-to-Income Ratio							
Data	0.2626	0.3511	0.5517	0.7301	0.8600	1.4551	1.5251
Model	0.2747	0.3549	0.5058	0.7134	0.9092	1.4593	1.4811
Panel B.2: Work Hours Share							
Data	0.4184	0.4190	0.4189	0.4210	0.4184	0.4107	0.3897
Model	0.4247	0.4154	0.4102	0.4028	0.4097	0.4091	0.4200
Panel C: Blended Group							
Panel C.1: Stock Market Participation Rate							
Data	0.4380	0.5610	0.6792	0.6893	0.7041	0.5171	0.4509
Model	0.4268	0.5856	0.6115	0.6594	0.6354	0.5169	0.5488
Panel C.2: Conditional Stock Share							
Data	0.3870	0.4240	0.4401	0.4632	0.4500	0.4201	0.4286
Model	0.3851	0.4188	0.4539	0.4847	0.4654	0.4380	0.3936
Panel C.3: Financial Wealth-to-Income Ratio							
Data	0.4996	0.6376	1.0294	1.6036	2.7920	3.2311	3.3764
Model	0.5542	0.7182	1.0067	1.3582	1.8377	3.1459	3.9484
Panel C.4: Work Hours Share							
Data	0.4184	0.4190	0.4189	0.4210	0.4184	0.4107	0.3897
Model	0.3946	0.3985	0.3995	0.3996	0.3994	0.3971	0.3933

Figure E.1: Matching Results: Non-Stockholder Model
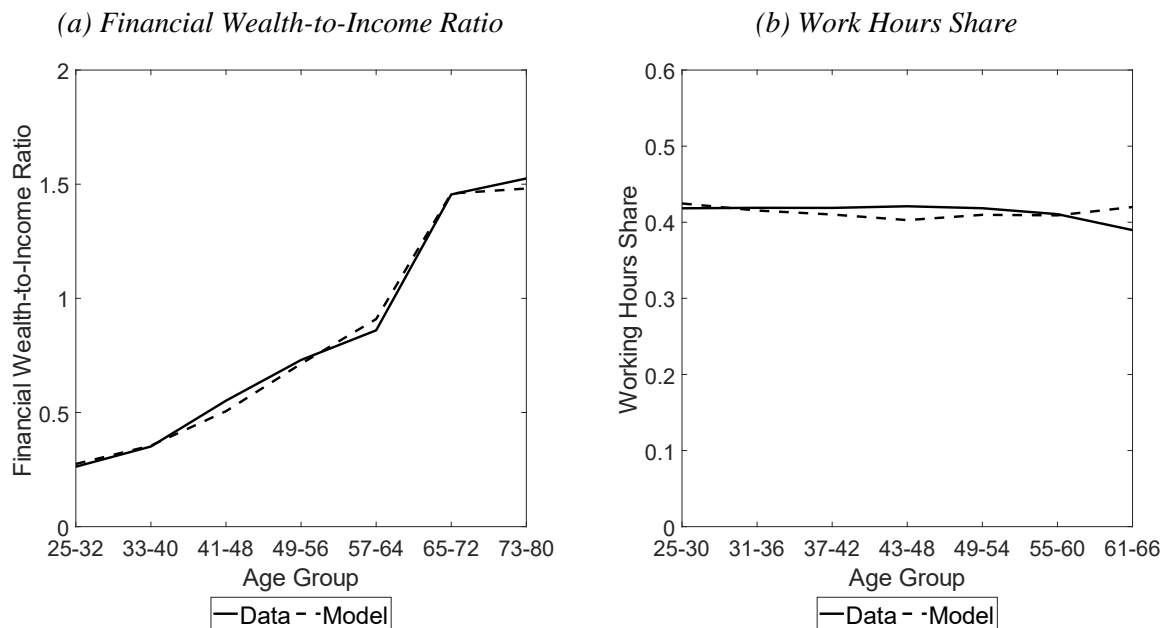
(a) Financial Wealth-to-Income Ratio   *(b) Work Hours Share*

Notes: This figure compares the fit of the moments implied by the structural estimation of the life-cycle model for only non-stockholders with the data moments. The model moments are the ones implied by the parameter combination that gives the best fit in the structural estimation for the respective preference specification. Those moments are compared to the empirical moments estimated by using SCF data (1989–2019) and PSID data (1975–2017), and by utilizing the methodology of Deaton and Paxson (1994). In the structural estimation the variables financial wealth-to-income ratio for non-stockholders and work hours share are simultaneously targeted. The model moments are the mean values from 100,000 simulated life cycles based on optimal feedback controls.

Figure E.2: Matching Results: Blended Group Model
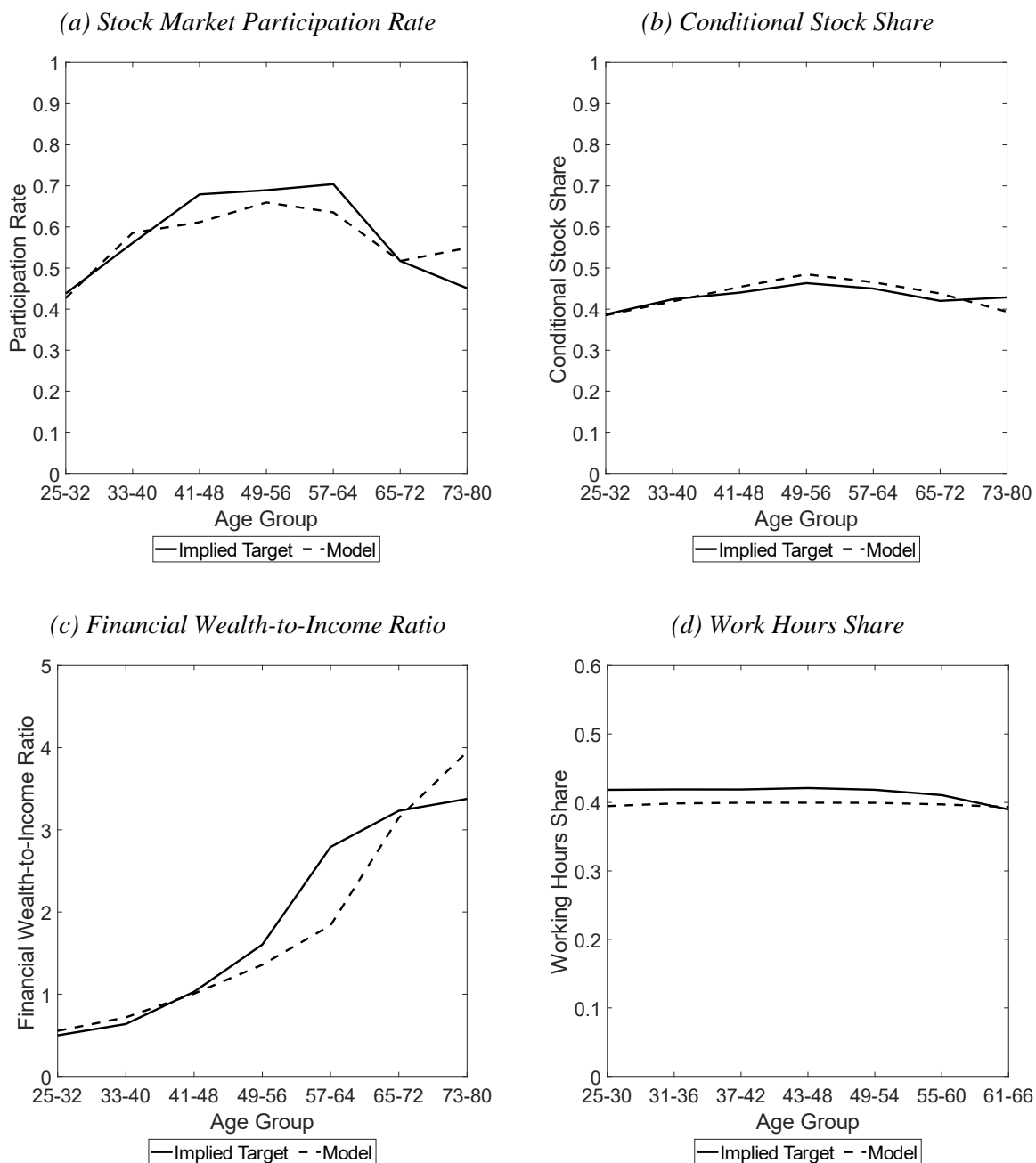
(a) Stock Market Participation Rate

(b) Conditional Stock Share

(c) Financial Wealth-to-Income Ratio

(d) Work Hours Share

Notes: This figure compares the fit of the moments implied by the structural estimation of the life-cycle using the blended group of the heterogeneous agent models with the data moments. Model moments are those implied by the parameter combination that gives the best fit in the structural estimation for the respective preference specification. Those moments are compared to the empirical moments estimated by using SCF data (1989–2019) and PSID data (1975–2017), and by utilizing the methodology of Deaton and Paxson (1994). In the structural estimation, the stock market participation rate, conditional stock share, financial wealth-to-income ratio, and work hours share are simultaneously targeted. The model moments are the mean `values from 100,000 simulated life cycles based on optimal feedback controls.

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